Using the NCTM 2000 PRINCIPLES AND STANDARDS *with the* LEARNING FROM ASSESSMENT *materials*

One of the key activities in Module 1: Aligning Assessment to Standards features a "deck" of standards from the NCTM ***Curriculum and Evaluation Standards for School Mathematics*** (1989). To update the materials and use the recently–released NCTM ***Principles and Standards for School Mathematics*** (2000), print the following on cardstock to create "decks" of NCTM 2000 ***Principles and Standards.*** (These replace **BLM 9a-e.**)

Excerpts from *PRINCIPLES AND STANDARDS FOR SCHOOL MATHEMATICS*, NCTM Standards 2000 Commemorative CD-ROM

PRINCIPLES AND STANDARDS FOR SCHOOL MATHEMATICS

Decisions made by teachers, school administrators, and other education professionals about the content and character of school mathematics have important consequences both for students and for society. These decisions should be based on sound professional guidance. *Principles and Standards for School Mathematics* is intended to provide such guidance. The Principles describe particular features of high-quality mathematics education. The Standards describe the mathematical content and processes that students should learn. Together, the Principles and Standards constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems.

These six Principles do not refer to specific mathematics content or processes and thus are quite different from the Standards. They describe crucial issues that, although not unique to school mathematics, are deeply intertwined with school mathematics programs. They can influence the development of curriculum frameworks, the selection of curriculum materials, the planning of instructional units or lessons, the design of assessments, the assignment of teachers and students to classes, instructional decisions in the classroom, and the establishment of supportive professional development programs for teachers. The perspectives and assumptions underlying the Principles are compatible with, and foundational to, the Standards and expectations presented in the document *Principles and Standards for School Mathematics.* The power of these Principles as guides and tools for decision making derives from their interaction in the thinking of educators. The Principles will come fully alive as they are used together to develop high-quality school mathematics programs.

The six principles for school mathematics address overarching themes:

Equity. Excellence in mathematics education requires equity—high expectations and strong support for all students.

Curriculum. A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

Teaching. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

Learning. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Assessment. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

Technology. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

NUMBER AND OPERATIONS

Instructional programs from prekindergarten through grade 12 should enable all students to—

A **Understand numbers, ways of representing numbers, relationships among numbers, and number systems.**
In grades 6–8 all students should—

- **A1** work flexibly with fractions, decimals, and percents to solve problems;
- **A2** compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line;
- **A3** develop meaning for percents greater than 100 and less than 1;
- **A4** understand and use ratios and proportions to represent quantitative relationships;
- **A5** develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;
- **A6** use factors, multiples, prime factorization, and relatively prime numbers to solve problems;
- **A7** develop meaning for integers and represent and compare quantities with them.

B **Understand meanings of operations and how they relate to one another.** *In grades 6–8 all students should—*

- **B1** understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- **B2** use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals;
- **B3** understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.

C **Compute fluently and make reasonable estimates.** *In grades 6–8 all students should—*

- **C1** select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods;
- **C2** develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use;
- **C3** develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results;
- **C4** develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

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ALGEBRA

Instructional programs from prekindergarten through grade 12 should enable all students to—

A **Understand patterns, relations, and functions.** *In grades 6–8 all students should—*

- **A1** represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- **A2** relate and compare different forms of representation for a relationship;
- **A3** identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

B **Represent and analyze mathematical situations and structures using algebraic symbols.**
In grades 6–8 all students should—

- **B1** develop an initial conceptual understanding of different uses of variables;
- **B2** explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;
- **B3** use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
- **B4** recognize and generate equivalent forms for simple algebraic expressions and solve linear equations.

C **Use mathematical models to represent and understand quantitative relationships.**
In grades 6–8 all students should—

- **C1** model and solve contextualized problems using various representations, such as graphs, tables, and equations.

D **Analyze change in various contexts.** *In grades 6–8 all students should—*

- **D1** use graphs to analyze the nature of changes in quantities in linear relationships.

GEOMETRY

Instructional programs from prekindergarten through grade 12 should enable all students to—

A Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. *In grades 6–8 all students should—*

- **A1** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects (e.g., angles, triangles, quadrilaterals, cylinders, cones) using their defining properties;
- **A2** understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects;
- **A3** create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

B Specify locations and describe spatial relationships using coordinate geometry and other representational systems. *In grades 6–8 all students should—*

- **B1** use coordinate geometry to represent and examine the properties of geometric shapes;
- **B2** use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.

C Apply transformations and use symmetry to analyze mathematical situations. *In grades 6–8 all students should—*

- **C1** describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling;
- **C2** examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

D Use visualization, spatial reasoning, and geometric modeling to solve problems.
In grades 6–8 all students should—

- **D1** draw geometric objects with specified properties, such as side lengths or angle measures;
- **D2** use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- **D3** use visual tools such as networks to represent and solve problems;
- **D4** use geometric models to represent and explain numerical and algebraic relationships;
- **D5** recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

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MEASUREMENT

Instructional programs from prekindergarten through grade 12 should enable all students to—

A Understand measurable attributes of objects and the units, systems, and processes of measurement.
In grades 6–8 all students should—

- **A1** understand both metric and customary systems of measurement;
- **A2** understand relationships among units and convert from one unit to another within the same system;
- **A3** understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.

B Apply appropriate techniques, tools, and formulas to determine measurements.
In grades 6–8 all students should—

- **B1** use common benchmarks to select appropriate methods for estimating measurements;
- **B2** select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision;
- **B3** develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more- complex shapes;
- **B4** develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders;
- **B5** solve problems involving scale factors, using ratio and proportion;
- **B6** solve simple problems involving rates and derived measurements for such attributes as velocity and density.

DATA ANALYSIS AND PROBABILITY

Instructional programs from prekindergarten through grade 12 should enable all students to—

A Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them. *In grades 6–8 all students should—*

- **A1** formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population;
- **A2** select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.

B Select and use appropriate statistical methods to analyze data. *In grades 6–8 all students should—*

- **B1** find, use, and interpret measures of center and spread, including mean and interquartile range;
- **B2** discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.

C Develop and evaluate inferences and predictions that are based on data. *In grades 6–8 all students should—*

- **C1** use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken;
- **C2** make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit;
- **C3** use conjectures to formulate new questions and plan new studies to answer them.

D Understand and apply basic concepts of probability. *In grades 6–8 all students should—*

- **D1** understand and use appropriate terminology to describe complementary and mutually exclusive events;
- **D2** use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations;
- **D3** compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

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PROBLEM SOLVING

Instructional programs from prekindergarten through grade 12 should enable all students to—

A Build new mathematical knowledge through problem solving;

B Solve problems that arise in mathematics and in other contexts;

C Apply and adapt a variety of appropriate strategies to solve problems;

D Monitor and reflect on the process of mathematical problem solving.

Through problem solving, students can experience the power and utility of mathematics. Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas. Middle-grades students whose curriculum is based on the Standards in this document will benefit from frequent opportunities for both independent and collaborative problem-solving experiences. They will engage profitably in complex investigations, perhaps occasionally working for several days on a single problem and its extensions.

Problem solving in grades 6–8 should promote mathematical learning. Students can learn about, and deepen their understanding of, mathematical concepts by working through carefully selected problems that allow applications of mathematics to other contexts. Many interesting problems can be suggested by everyday experiences, such as reading literature or using cellular telephones, in-line skates, kites, and paper airplanes.

Instruction in grades 6–8should take advantage of the expanding mathematical capabilities of students to include more-complex problems that integrate such topics as probability, statistics, geometry, and rational numbers. Situations and approaches should build on and extend the mathematical understanding, skills, and language that students have acquired. Well-chosen problems can be particularly valuable in developing or deepening students' understanding of important mathematical ideas.

Teachers should regularly ask students to formulate interesting problems based on a wide variety of situations, both within and outside mathematics. Teachers should also give students frequent opportunities to explain their problem-solving strategies and solutions and to seek general methods that apply to many problem settings. These experiences should engender in students important problem-solving dispositions—an orientation toward problem finding and problem posing; an interest in, and capacity for, explaining and generalizing; and a propensity for reflecting on their work and monitoring their solutions. They should be expected to explain their ideas and solutions in words first, and then teachers can help them learn to use conventional mathematical symbols or their own forms of representations, as appropriate, to convey their thinking.

REASONING AND PROOF

Instructional programs from prekindergarten through grade 12 should enable all students to—

A Recognize reasoning and proof as fundamental aspects of mathematics;

B Make and investigate mathematical conjectures;

C Develop and evaluate mathematical arguments and proofs;

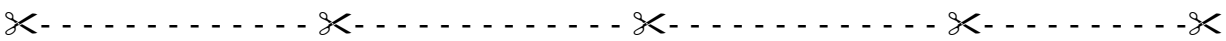
D Select and use various types of reasoning and methods of proof.

Reasoning is an integral part of doing mathematics. Students should enter the middle grades with the view that mathematics involves examining patterns and noting regularities, making conjectures about possible generalizations, and evaluating the conjectures. In grades 6–8 students should sharpen and extend their reasoning skills by deepening their evaluations of their assertions and conjectures and using inductive and deductive reasoning to formulate mathematical arguments. They should expand the audience for their mathematical arguments beyond their teacher and their classmates. They need to develop compelling arguments with enough evidence to convince someone who is not part of their own learning community.

In the middle grades, students should have frequent and diverse experiences with mathematics reasoning as they—

- examine patterns and structures to detect regularities;
- formulate generalizations and conjectures about observed regularities;
- evaluate conjectures;
- construct and evaluate mathematical arguments.

Students should discuss their reasoning on a regular basis with the teacher and with one another, explaining the basis for their conjectures and the rationale for their mathematical assertions. Through these experiences, students should become more proficient in using inductive and deductive reasoning appropriately. As examples, students can use inductive reasoning to search for mathematical relationships through the study of patterns. And, middle-grades students can develop arguments to support their conclusions in varied topics, such as number theory, properties of geometric shapes, and probability.



COMMUNICATION

Instructional programs from prekindergarten through grade 12 should enable all students to—

A Organize and consolidate their mathematical thinking through communication;

B Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;

C Analyze and evaluate the mathematical thinking and strategies of others;

D Use the language of mathematics to express mathematical ideas precisely.

In classrooms where students are challenged to think and reason about mathematics, communication is an essential feature as students express the results of their thinking orally and in writing. This type of environment is desirable at all grade levels, but there are a few distinctive features of such classrooms in the middle grades. For example, the mathematics under discussion in grades 6–8 is generally more complex and perhaps more abstract than the mathematics in the lower grades.

A second distinctive feature relates to the norms for evaluating the thinking of members of a classroom learning community. When students in grades 6–8 explain their thinking, they can be held to standards that are more stringent than would likely be applied to younger students, though not as demanding as might be applied in high school. Each student should be expected not only to present and explain the strategy he or she used to solve a problem but also to analyze, compare, and contrast the meaningfulness, efficiency, and elegance of a variety of strategies. Explanations should include mathematical arguments and rationales, not just procedural descriptions or summaries (Yackel and Cobb 1996).

A third distinguishing feature pertains to the social norms in a middle-grades classroom rather than to the content of the students' discussions. During adolescence, students are often reluctant to do anything that causes them to stand out from the group, and many middle-grades students are self-conscious and hesitant to expose their thinking to others. Peer pressure is powerful, and a desire to fit in is paramount. Teachers should build a sense of community in middle-grades classrooms so students feel free to express their ideas honestly and openly, without fear of ridicule.

CONNECTIONS

Instructional programs from prekindergarten through grade 12 should enable all students to—

A Recognize and use connections among mathematical ideas;

B Understand how mathematical ideas interconnect and build on one another to produce a coherent whole;

C Recognize and apply mathematics in contexts outside of mathematics.

Thinking mathematically involves looking for connections, and making connections builds mathematical understanding. Without connections, students must learn and remember too many isolated concepts and skills. With connections, they can build new understandings on previous knowledge. The important mathematical foci in the middle grades—rational numbers, proportionality, and linear relationships—are all intimately connected, so as middle-grades students encounter diverse new mathematical content, they have many opportunities to use and make connections.

Mathematics classes in the middle grades should continually provide opportunities for students to experience mathematics as a coherent whole through the curriculum used and the questions teachers and classmates ask. Students reveal the ways they are connecting ideas when they answer questions such as, What made you think of that? Why does that make sense? Where have we seen a problem like this before? How are these ideas related? Did anyone think about this in a different way? How does today's work relate to what we have done in earlier units of study? From these discussions, students can develop new connections and enhance their own understanding of mathematics by listening to » their classmates' thinking.

If curriculum and instruction focus on mathematics as a discipline of connected ideas, students learn to expect mathematical ideas to be related. Rich mathematical tasks prompt students to use and develop mathematical understandings and connections. Challenging problems encourage students to think about how familiar concepts and procedures can be applied in new situations. In classrooms where students are expected to reason mathematically and to communicate clearly about significant mathematical tasks, new ideas surface quite naturally as extensions of previously learned mathematics. With prompting from their teacher, students routinely ask themselves, "How is this problem like what I have done before? How is it different?"

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REPRESENTATION

Instructional programs from prekindergarten through grade 12 should enable all students to—

A Create and use representations to organize, record, and communicate mathematical ideas;

B Select, apply, and translate among mathematical representations to solve problems;

C Use representations to model and interpret physical, social, and mathematical phenomena.

Representation is central to the study of mathematics. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations. Representations—such as physical objects, drawings, charts, graphs, and symbols—also help students communicate their thinking.

Representations are ubiquitous in the middle-grades mathematics curriculum proposed here. The study of proportionality and linear relationships is intertwined both with students' learning to use variables flexibly in order to represent unknowns and with their learning to employ tables, graphs, and equations as tools for representation and analysis. Middle-grades students who are taught with this Standard in mind will learn to recognize, compare, and use an array of representational forms for fractions, decimals, percents, and integers. They also will learn to use representational forms such as exponential and scientific notation when working with large and small numbers and to use a variety of graphical tools to represent and analyze data sets.

Students in the middle grades solve many problems in which they create and use representations to organize and record their thinking about mathematical ideas. For example, they use representations to develop or apply their understanding of proportionality when they make or interpret scale drawings of figures or scale models of objects, when they connect the geometric notion of similarity with numerical ratios, and when they draw relative-frequency histograms for data sets. While solving challenging problems, students might use standard representations, but they can also develop and use nonstandard representations that work well for a particular problem. The study of linear functions, with the associated patterns and relationships, is another major focus in the middle grades. By considering problems in a variety of contexts, students should become familiar with a range of representations for linear relationships, including tables, graphs, and equations. Students need to learn to use these representations flexibly and appropriately. Students will be better able to solve a range of algebra problems if they can move easily from one type of representation to another. In the middle grades, students often begin with tables of numerical data to examine a pattern underlying a linear function, but they should also learn to represent those data in the form of a graph or equation when they wish to characterize the generalized linear relationship. Students should also become flexible in recognizing equivalent forms of linear equations and expressions. This flexibility can emerge as students gain experience with multiple ways of representing a contextualized problem.

Finally, it is important that middle-grades students have opportunities to use their repertoire of mathematical representations to solve relatively large-scale, motivating, and significant problems that involve modeling physical, social, or mathematical phenomena. The goal of this sort of mathematical modeling is for students to gain experience in using the mathematics they know and an appreciation of its utility for understanding and solving applied problems.