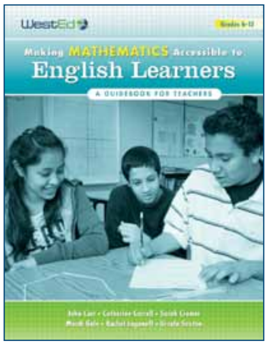

Chapter 1 — Teaching Mathematics

from Making Mathematics Accessible to English Learners: A Guidebook for Teachers

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CHAPTER 1

Teaching Mathematics

This chapter offers an overview of mathematics instruction that is effective for diverse learners, particularly English learners. We begin with three principles of learning and motivation that apply to all learners. Then we describe an inquiry-based approach to mathematics instruction that has fidelity with National Council of Teachers of Mathematics (NCTM) standards.[1] We end the chapter with a description of how to use various modes of instruction for English learners.

PRINCIPLES OF LEARNING AND MOTIVATION

Regardless of whether students are native English speakers or English learners, three research-based principles about how people learn[2] guide effective mathematics teaching and learning. These principles are the foundation of all of the ideas and strategies presented in this guidebook. Making mathematics accessible to English learners means, first of all, recognizing how any student learns.

> **Principle 1.** Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.
>
> *English learners, like any learners, need a way to connect what they know with what they need to learn.*
>
> **Principle 2.** To develop competence in an area of inquiry, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.
>
> *English learners, like any learners, need to learn facts and ideas and need to be able to relate and organize them conceptually.*
>
> **Principle 3.** A metacognitive approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.
>
> *English learners, like any learners, benefit from reflecting on their learning goals and progress. English learners, unlike native English speakers, will need to apply a metacognitive approach to learning English, as well as to learning discipline-specific content — in this case, mathematics content.*

The artful teacher brings these principles to life for each student, recognizing a student's current level of knowledge and understanding and facilitating each student's growth as a self-directed learner. A respectful classroom climate is key to a teacher's success in being able to do this.[3] Often a visitor can step into a classroom and feel a distinct climate, whether of respect and caring, fear of ridicule, or boredom and detachment. A positive climate is established by teacher modeling and facilitation and is sustained by student practice.

When teachers nurture a safe learning community within their classrooms, students respect each other's ideas, are patient with one another, recognize there can be multiple perspectives and ways of learning, and recognize the value of individual contributions to group learning. With their anxiety lowered, students are physiologically more able to accept new challenges and grapple with new concepts and problems.[4] Because English learners can be expected to feel high levels of anxiety about all the challenges they face, it is especially important for them to feel respected by the teacher and other students, whether they are struggling to learn English and mathematics or to communicate different cultural perspectives they may bring to discussions.

Within inclusive classrooms, educators increasingly recognize that equitable and equal are not synonymous. Widespread interest in differentiating instruction reflects the understanding that students learn in different ways. Providing a high-quality mathematics education for all students means planning and using strategies that fit diverse students. Inquiry-based mathematics education meets these goals for providing equitable access to the curriculum for all students, including English language learners.[5]

AN INQUIRY-BASED APPROACH TO MATHEMATICS EDUCATION

When taught well, mathematics requires students to solve problems, reason, and communicate. Effective mathematics teaching "engage[s] all students in:

» Formulating and solving a wide variety of problems

» Making conjectures and constructing arguments

» Validating solutions

» Evaluating the reasonableness of mathematical claims"[6]

The instructional strategies described in this guidebook offer teachers a way to involve English learners in an inquiry-based, language-rich approach to solving mathematical problems. An inquiry-based approach to mathematics instruction for English learners explicitly targets content and language objectives, and is implemented through various instructional methods, including student inquiry and teacher modeling.[7] Inquiry-based instruction is rooted in constructivist learning theory and capitalizes on students' curiosity by offering hands-on, *authentic* activities and tasks (Principle 1 of learning and motivation). An authentic problem is one whose context is interesting or meaningful to students, whether they are working on a word problem connected to real-world experiences or a purely mathematical problem.[8]

Teachers begin lesson planning for inquiry-based mathematics instruction by identifying key mathematics standards and related mathematical ideas (Principle 2 of learning and motivation). Students systematically acquire knowledge as lessons progress through these connected or overlapping mathematical ideas. During classroom instruction, the teacher models mathematical thinking and guides students' understanding of the interrelationships among mathematical ideas. Throughout the learning process, the teacher monitors student thinking and involves students in reflecting on their own thinking (Principle 3 of learning and motivation).

To introduce a mathematical idea, the teacher poses an engaging, intellectually challenging task or problem. Students are then guided to use mathematical reasoning to reach conclusions about the task; to justify their conclusions; and to generalize about them.[9] The teacher also instructs students in such cognitive tasks as judging effects of mathematical operations, understanding mathematical properties, and making connections among mathematical concepts. As part of this cognitive guidance, the teacher models mathematical language and appropriate ways to discuss mathematical problems.[10] As one teacher remarked, "All of my students — English learners and native speakers — need to learn the language of mathematics and use that language to discuss their mathematics reasoning about real problems."[11] The development of these skills builds the foundation for and supports effective social learning.

Social learning[12] is an integral part of inquiry-based learning and can be effective for English learners[13] when the teacher models target language and discussion expectations, provides visual language supports (e.g., Word Walls, Sentence Starters), and monitors and guides students as they work collaboratively in small groups. Social learning is effective because it promotes a rich environment for the use of academic language, such as problem-solving discussions that require use of mathematical terminology. Students are expected to practice purposeful listening and speaking skills. As they work in small groups, English learners hear their peers rephrase what the teacher has said and discuss their ideas, with the support of visuals and hands-on activities.[14] During small-group activities, the teacher monitors students' academic, social, and English language learning as they propose and try alternative problem-solving strategies, and explain their conceptual thinking through speaking and writing. In a supportive classroom, English learners benefit from a variety of such collaborations, which offer repeated opportunities to participate in discourse that builds their mathematical and literacy knowledge.[15]

In spite of what we know about effective instructional practices, mathematics continues to be taught in some classrooms as it has been traditionally: As a fixed body of knowledge and set of procedures. Students are asked to reproduce mathematical expressions, but are rarely expected to produce innovative solutions to mathematical problems. As a result, students achieve automaticity in reproducing mathematical expressions or performing computations, but do not develop *mathematical literacy*. Individuals are considered mathematically literate when they can use mathematics as a fully functioning member of a society.[16] This includes the ability to read and understand mathematics content in newspaper articles (e.g., pie charts, line graphs, data tables, averages, percentages, and sampling error in polls) and use mathematics in everyday tasks.

When instruction focuses on having students simply manipulate mathematical expressions and practice algorithms, it avoids the important cognitive challenges of understanding word problems and discussing mathematical ideas. This type of approach is generally not effective for any learner, but it is especially problematic for English learners because it does not involve them in the mathematical thinking and talking that support both language development and mathematics learning.[17] Specifically, they do not learn to identify the important facts in a real-world problem, select an appropriate strategy to solve the problem, and explain their reasoning. The ultimate consequence is that English learners become marginalized in mathematics education and do not have the opportunity to become mathematically literate or choose a math-oriented career.

To elaborate on the recommendations for English learners in mathematics classrooms, we present a list of best instructional practices for English learners in mainstream mathematics classrooms:[18]

» Provide a rich, meaning-centered context for students to use language, with many visual representations, hands-on activities, and language supports.

» Provide ample opportunities for high-quality interaction between English learners and native English speakers that encourage English learners to share their knowledge and experiences, hear other students rephrase what the teacher said, and apply new language.

» Use high-frequency vocabulary that students know and gradually introduce more academic vocabulary as they progress in the lesson and their language skills.

» Integrate listening, speaking, reading, and writing skills across instruction, and assist English learners to make a bridge between oral and written language.

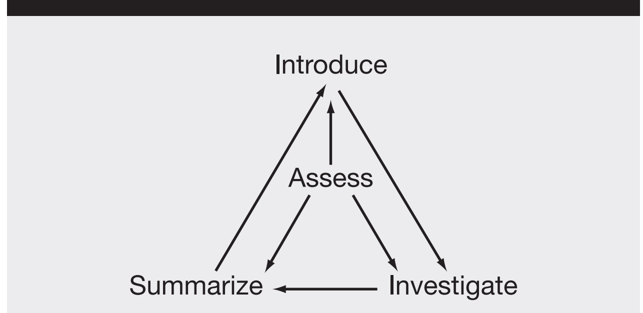
In order to help students advance as learners of mathematics and of the English language, these strategies blend direct teaching and student inquiry,[19] and are implemented in the context of high-quality mathematics instruction.

THE 3-PHASE MODEL OF TEACHING AND LEARNING MATHEMATICS

Although the mathematics community has not reached consensus about what to call them, it is common practice among research-based mathematics curricula to organize lessons into three phases.[20] During the first phase, often called "introduce" or "launch," the teacher encourages students to draw on their prior knowledge in order to engage with a new concept. In phase two, "investigate" or "explore," students work with the new concept in the form of a meaningful problem. During the third phase of a mathematics lesson, "summarize" or "wrap-up," students and teachers draw conclusions and make connections to related concepts.

In this guidebook, we use *introduce*, *investigate*, and *summarize* to label the three phases, as reflected in Figure 1.1. Note that student assessment is continuous throughout the three phases because teachers use feedback from assessment to adjust instruction during all phases. Each of the three phases is described below,[21] followed by an excerpt[22] of a teacher's vision for implementing that phase in the classroom.

FIGURE 1.1. **Three Phases of Mathematics Instruction**



Introduce

The learning process begins as the teacher guides students to make connections between the learning task at hand and their past academic, personal, and cultural experiences. The goal is to engage students in learning by sparking their curiosity, posing intriguing problems, or asking thought-provoking questions. This phase also offers the teacher opportunities to identify students' preconceptions and misconceptions about a mathematical concept. When misconceptions arise, they are simply acknowledged along with other brainstorming ideas, but the teacher mentally notes these misunderstandings to ensure that they are explicitly addressed at the proper time.

As part of this phase, it can also be useful for a teacher to make explicit the mathematics and language objectives that are the focus of the lesson. For example, as part of the SIOP® model,[23] the teacher presents these objectives to students orally and in writing. Doing so makes it crystal clear to students how the planned discussions and activities will address mathematics standards and English language standards.[24] (Language objectives specific to a lesson may be derived from charts of Academic Language Skills that are presented in chapter 3.) When the teacher makes learning objectives explicit, it helps all students focus on the "bull's eye" from the start of the lesson; and it sets the basis for students to reflect on how well they achieved those objectives at the end of the lesson. The teacher plans a lesson that targets those specific content and language objectives, and reflects after the lesson on how well the instructional strategies and learning activities stayed on course and met the objectives.

> **In my classroom . . .** I begin my lesson plan with an intriguing idea, image, or question to engage students. I pose questions about what my students already know, make conjectures about how to solve a problem, and encourage students to pose questions about what they want to learn. This alerts me to what students already know, their misconceptions, and areas of potential confusion. I let students know at the start of each lesson what our mathematics and English language objectives are so that students understand the purpose of the activity.

Investigate

The teacher guides students as they investigate a mathematical task, work toward a common understanding of specific concepts, and acquire problem-solving and computational skills. The teacher designs activities that encourage students to construct new knowledge or skills, propose preliminary ways of thinking about a problem, "puzzle" through problems, and try alternatives to reach a solution. As students engage with the mathematics, the teacher encourages them to demonstrate or explain their conceptual understanding of the problem and the process skills they used to arrive at their conclusion. Students debate alternative explanations for their conclusions and use new facts to correct their prior misconceptions. As appropriate, the teacher directs students' attention back to helpful points from the introduce phase of instruction. Students are guided to organize information supporting their ideas or conclusions into evidence-based statements, using mathematical language.

> **In my classroom . . .** Rather than telling students the concepts I want them to learn, I expect them to think critically about the concepts by experimenting, investigating, observing, classifying, communicating, measuring, predicting, and interpreting. This active engagement arouses their curiosity and leads them to discover new ideas, confirm prior assumptions, or reconsider their earlier thinking.
>
> I guide students to explain their thinking by asking questions and facilitating peer discussions. I give students time to think, and I facilitate student–student discussions to correct misconceptions. I provide time to question and justify answers. I do not just answer questions that students pose, nor do I simply decide for them which answers are right or wrong. By

listening to their ideas and reasoning, I can determine the next instructional experiences I want to provide students.

Summarize

The summarizing phase involves more than just revisiting what has been learned. During this phase, the teacher engages students in activities and discussions that challenge and extend their conceptual understanding and problem-solving skills. Students apply what they have learned to new mathematical tasks and experiences to develop, extend, connect, and deepen their understanding of the concepts.

> **In my classroom . . .** At the end of an instructional unit, I help students compare, contrast, combine, synthesize, generalize, and make inferences by asking them to solve a problem or perform a task that introduces a somewhat different context from those they have just experienced. I want students to be able to apply new knowledge, make connections, and extend ideas. Their various ideas for applying their knowledge help me to differentiate instruction better so that all students can engage in activities.

Assess

Throughout the three phases of inquiry-based mathematics instruction, the teacher assesses students' progress and asks students to evaluate themselves. Feedback may come from quick, on-the-spot checks for understanding (e.g., expressed with hand gestures, white boards), quizzes, student discussions, journals, or other techniques. The teacher uses the feedback to reflect on how effective a lesson was, and to make mid-lesson adjustments to better meet students' needs and interests. Students use the feedback to reflect on what they understand, what they still need to learn, and what they want to learn next. (See chapter 6 for a fuller discussion of assessment strategies and uses.)

> **In my classroom . . .** I test students on more than just factual knowledge; during an assessment, I challenge students to construct ideas and explanations, just as I do during classroom instruction. I want assessments to reflect both my objectives and the content standards.

Throughout each phase of mathematics instruction for English learners, the role of the teacher is multifaceted and ever changing. As a facilitator, the teacher nurtures creative thinking, problem solving, interaction, communication, and discovery. As a model, the teacher initiates thinking processes, inspires positive attitudes toward learning, motivates, and demonstrates skill-building techniques and the effective use of language to communicate mathematical thinking. Finally, as a guide, the teacher helps to bridge language gaps and foster individuality, collaboration, and personal growth. The teacher moves flexibly into and out of these various roles, as appropriate for each lesson.

Figure 1.2 identifies aspects of what the teacher and the student may be doing in each of the three instructional phases. Assessment methods (which appear in italics) encourage students to assess their understanding and abilities, and they provide opportunities for teachers to evaluate student progress. All teacher and student activities listed below are especially important for English learners.

FIGURE 1.2. **Three-Phase Model for Teaching and Learning Mathematics**

Purpose	How the Teacher Is Engaged	How Students Are Engaged
Introduce		
To initiate the lesson: » Connect students' past and present learning experiences » Anticipate new ideas » Focus students' thinking toward the mathematical goals of the lesson. Mathematics and language objectives are stated orally and posted on the wall.	» Create interest and generate curiosity » Raise questions and problems » Ensure students understand the mathematical purpose of the activity » Elicit responses that uncover students' current knowledge about the concept/topic » *Refer students to existing data and evidence and ask, "What do you already know?" "Why do you think…?"**	» Asking questions such as "What do I already know about this question?" "What are the parts of the question that I need to use to answer the question?" "What problem-solving strategies can I use to answer the question?" » Showing interest in the topic » *Discussing understanding with peers*
Investigate		
To provide students with experiences within which mathematical concepts, processes, and skills are identified and developed.	» Guide students to work together » Observe and listen to students as they interact » Provide time for students to puzzle through problems » Redirect students' investigations as needed » Guide students to explain concepts and definitions in their own words » Suggest definitions, explanations, and new vocabulary » Elicit mathematical justification from students » *Observe and listen to students as they apply new concepts and skills* » *Ask open-ended questions such as "What evidence do you have?" "How did you think about the question?"*	» Thinking creatively within the limits of the task » Forming, testing, and refining conjectures and strategies » Trying alternatives to solve a problem and discussing them with others » Explaining possible solutions or answers to other students » Generating definitions and explanations of concepts » Listening critically to and respectfully questioning explanations from the teacher and from other students » Connecting to previous activities » *Answering open-ended questions by using observations, evidence, and previously accepted explanations*

* Italics indicate assessment methods.

FIGURE 1.2. **Three-Phase Model for Teaching and Learning Mathematics (continued)**

Purpose	How the Teacher Is Engaged	How Students Are Engaged
Summarize		
To make mathematical connections and extend students' conceptual understanding and skills.	» Help students understand alternative explanations and make connections among mathematical ideas » Encourage students to apply the concepts and skills in new situations » *Assess students' conceptual understanding and knowledge/ skills* » *Look for evidence that students have changed their thinking* » *Look for evidence that students are deepening their understanding*	» Applying new labels, definitions, explanations, and skills in new but connected situations » Drawing on experience to ask questions, propose solutions, make decisions, and present reasonable conclusions based on evidence » *Demonstrating an understanding or knowledge of the concept or skill* » *Evaluating own progress and knowledge* » *Asking related questions that would encourage future investigations*

* Italics indicate assessment methods.

This three-phase approach to mathematics lessons is related to a similar approach in science education. Our broad intent is for mathematics and science teachers to see commonalities in the mathematics and science guidebooks, and to collaborate on instructional practices. The three instructional phases in mathematics education correspond to the five stages of inquiry-based science instruction described in Roger Bybee's "5 Es" model (see Figure 1.3). A description of the teacher's role during each of the five stages follows.

1. Engage: Activates students' prior knowledge and preconceptions, relates concepts to students' interests
2. Explore: Guides students to investigate a phenomenon
3. Explain: Supports students to identify and discuss explanations for scientific phenomena
4. Elaborate: Guides students to apply learned concepts to new experiences to extend and deepen understanding
5. Evaluate: Evaluates students' progress and adjusts instruction to fit student needs.[25]

Both instructional models can provide a deeper understanding of the teacher's role in an inquiry-based classroom. The 5 Es of science instruction are more fully elaborated in the research than are the three phases for mathematics lessons; Appendix A includes a full description of this model for inquiry-based science teaching and learning.

FIGURE 1.3. **Mathematics Phases and Science Stages**

3 PHASES IN MATHEMATICS	5 Es IN SCIENCE	
Introduce	Engage	Evaluate
Investigate	Explore	
	Explain	
Summarize	Elaborate	

Source: Carr, J., Sexton, U., & Lagunoff, R. (2007). *Making Science Accessible to English Learners: A Guidebook for Teachers*. San Francisco, CA: WestEd.

THREE MODES OF INSTRUCTION APPLIED TO ENGLISH LEARNERS

Throughout the phases of mathematics instruction, teachers commonly combine three modes of instruction — teacher-directed, teacher-assisted, and peer-assisted (see Figure 1.4). In a mathematics classroom, each of these modes offers students distinct learning benefits and opportunities. Adjustments to each mode can further enhance English learner engagement and understanding.

Teacher-directed. English learners will feel comfortable during teacher-directed instruction when the teacher provides comprehensible input,[26] using language and speech students can understand, and supporting it with visuals and demonstrations.

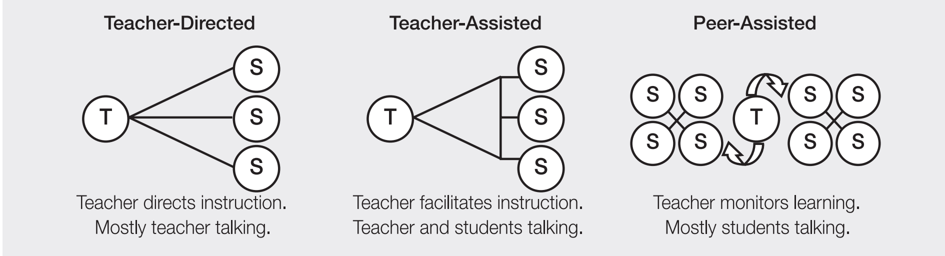
Teacher-assisted. English learners will feel more comfortable speaking in teacher-assisted conversations when the teacher establishes a risk-free, caring climate and takes students' proficiency levels into account when expecting them to comprehend input and produce meaningful output.

Peer-assisted. English learners will feel comfortable in peer-assisted instruction when respected and supported by peers in the group and when group tasks are within their communication capabilities. Peer-assisted instruction is an opportunity for English learners both to use their native academic language (if grouped by same language) and to participate as English listeners and speakers as key concepts are repeated and rephrased in English during whole-class discussion.

Teacher-Directed Instruction

In teacher-directed instruction, the teacher provides direct instruction to the whole class, and individual students respond to the teacher; most interactions are teacher–student. The teacher initiates concept development by giving direct instruction, demonstrating to the whole class, and modeling specific mathematical protocols and expected behaviors and processes. The teacher combines saying with showing — supporting oral instruction with pictures, illustrations, manipulatives, relevant objects, graphic organizers, models, demonstrations, video clips, and other visuals. Teacher-directed instruction gives students access to the information they need to process and manipulate ideas, clarify concepts, and build the connections among concepts that lead to greater understanding of mathematics.

FIGURE 1.4. **Three Modes of Instruction**



APPLICATIONS FOR ENGLISH LEARNERS

Teacher modeling is an important support for English learners. English learners need to preview completed projects and writing assignments; some students may never have had formal schooling or participated in similar tasks in their former countries. Similarly, before students engage in teacher-assisted discussions and peer-assisted learning activities, the teacher needs to model expected discourse and social behaviors and procedures.

When speaking to students, the teacher can target the listening abilities of English learners in terms of enunciation, speed, intonation, and use of vocabulary, idioms, and contractions. For example, English learners at novice levels require simple subject-verb-object sentences, free of idioms and colloquial expressions. (It can be surprising how many expressions such as “no way,” or “make up your mind,” or even “take a seat” pepper typical classroom instruction, to the befuddlement of English learners.)

For the benefit of English learners as well as other students, all teacher-directed instruction should be divided into chunks no longer than 15 minutes. Students need time to process each chunk before encountering another chunk;[27] they must relate new information to their prior knowledge and experiences, address any prior misunderstandings, and construct new understandings. English learners may also need extra time to process spoken information, so they can listen in English, think in their primary language, and translate their thinking back into English. In some instances they may also want to discuss unfamiliar English vocabulary and new mathematics concepts with someone who speaks their primary language.

Between chunks of presented material and information, the teacher checks for students’ understanding. Research indicates that learning improves for the whole class and the achievement gap narrows when the teacher uses techniques to get feedback about each student’s understanding during direct instruction and immediately makes appropriate adjustments.[28]

Calling on individual students is a way for the teacher to collect feedback from a few students at a time. Alternative ways to check for understanding can provide a broader range of information and encourage all students to respond:

» Students use white boards to write and display short answers.

- » Students signal agreement/disagreement/confusion with a point of view, solution, or approach, using colored cards or hand gestures.
- » Students answer chorally.
- » Students work in teams to respond. For example, teams contribute to a collective class solution to a problem; or partners talk with one another in English or their primary language before reporting to the class in English.

When asking a question of the whole class, the teacher waits several seconds so that all students have ample opportunity to process the question and think about an answer (three to seven seconds depending on the difficulty of the question). The teacher may acknowledge early hand raisers with a nod while still waiting to give everyone time to think of an answer. This wait time allows English learners to decipher the question, think, and formulate an answer in English. It also encourages more students to respond. To resist the urge to keep the lesson pace moving rapidly, the teacher may use a technique such as counting silently or pacing the floor one step per second. When the teacher calls on a student, walking close to the responding student may lessen the student's anxiety about speaking in front of many peers. When English learners hear other students rephrase information during responses, it provides additional opportunities to learn mathematics vocabulary and comprehend important ideas.

Teacher-Assisted Instruction

In teacher-assisted instruction, the teacher guides brainstorming and discussion among the whole class, through interactions that are student–student and teacher–student. For example, in the Think-Pair-Share activity, pairs of students quickly share an idea based on the teacher's question; then the teacher facilitates whole-group sharing and discussion of students' responses. (See chapter 5 for strategies to facilitate and scaffold student work and discussion.) Grouping students in different ways will help support students' language development and confidence in the classroom. (Figure 1.5 illustrates one teacher's grouping decisions.)

In this mode, the teacher does not lecture and give answers. When a student asks a question, other students respond rather than the teacher. Small groups or the whole class agree that an answer is correct, appropriate, or applicable based on supporting evidence and reasoning. Teacher-assisted instruction empowers and guides students to think and talk as mathematician apprentices. Through thoughtful questioning techniques, the teacher facilitates and probes to encourage critical thinking; responds to student questions with meaningful questions that engage them in further dialogue with each other and with the teacher; and promotes the revision or review of their interpretations based on the evidence at hand. As students construct meaning from their explorations and text, the teacher may deem it necessary to intervene — by providing further evidence, raw data, or other resources or interactions — so as to model closer examination of evidence to correct misconceptions. By providing a safe environment in which to express ideas, the teacher allows students to consider alternative interpretations and test new ideas, while continuing to build understanding based on empirical and quantitative evidence.

FIGURE 1.5. **One Teacher's Grouping Decisions**

Following is a brief scenario of a high school mathematics classroom where the teacher shifts among teacher-directed, teacher-assisted, and peer-assisted instruction. This scenario shows the advantages of grouping students by their primary language. Other grouping criteria should be used as well (e.g., social characteristics, topic choice, mathematics literacy).

Classroom setting. I speak only English and I have 35 students in my class, most of whom are English learners, representing five languages (Spanish, Vietnamese, Mandarin, Tagalog, and Russian).

Grouping. Often I plan flexible student groupings for mathematics tasks, mixing students by primary language, English literacy, and/or mathematics literacy. Other times I allow students to select their own groups, and they usually select friends who speak their language. Today, I allow students to self-select, and most do so by their primary language.

Modeling. I start by posing an authentic context to the class. I give directions to be sure students understand the expectations of the task and model some of the discourse I expect of students involved in this task. I point to sentence starters on the wall that scaffold English learners to articulate their thoughts (e.g., "I agree with ___ that…"; "What if we…"; and "I think that…").

Group learning. Then student groups talk about and complete their tasks. Discussions within the self-selected groups of English learners are typically a mix of English and primary language, depending on the group members' needs and comfort levels. For example, a group of English learners mixes Spanish and English during their discussion, with the more advanced English learners rephrasing certain ideas in Spanish for a very limited English learner who is showing difficulty understanding the English. I walk around to answer questions and ensure that they are all learning successfully. Later, when I model the English responses to the questions, all students — even the most recently immigrated English learners — must write in English. Later, when we have the full-class discussion in English, the most limited English proficient learners will have a good idea of what is being said because it was first discussed within the homogeneous language group.

The advantage to having students use their language of choice for peer-assisted learning and problem solving is that the focus stays on the mathematics content. Students are not inhibited by their varying abilities to communicate in English, so I feel more comfortable that they can really understand the concepts in the day's activity. However, many times I form heterogeneous groups by mixing more proficient and limited English learners with different primary languages so students must use only English to convey their ideas, and the more proficient English learners and native English speakers are models for the limited English learners.

Discussing. Next, I lead a whole-class discussion in English about what they did and said. I use the same structured format every time: (1) What strategy did you use to solve the problem? (2) Did you discover more than one strategy to solve it? and (3) How are the strategies that have been presented similar/different? I write students' answers as English sentences, projected so that they all can see. Students copy the sentences in their notebooks in English and make connections to their own solution notes. I find that repetition of common questions helps orient my English learners and provides a familiar context.

My newest student, who is at the *beginning* English learner level, benefits from listening to good oral models about content that is both familiar and meaningful, although I do not expect him to fully comprehend all that other students are saying. I assist other English learners to communicate their ideas by providing vocabulary, cues, and other structures that help them convey their thinking. The more proficient the English learner, the more elaborate I expect his or her comments to be. I do not ask "dumbed-down" questions, but I do adjust questions to be comprehensible for my English learners. When they finish responding, I selectively rephrase answers to model mathematics discourse and incorporate key vocabulary. This mathematical rephrasing benefits all students in the class.

Source: McCall-Perez, Z. (2005). *Grouping English learners for science*. Unpublished manuscript. Adapted with permission.

APPLICATIONS FOR ENGLISH LEARNERS

The teacher ensures that English learners can participate in a variety of ways. First, the teacher frequently combines manipulatives or visuals, such as word walls (see chapter 5), with teacher talk, emphasizing key words and concepts. English learners can easily glance at word walls to find words they want to use when they answer questions or participate in class discussions. Second, the teacher uses controlled speech, tailoring the wording of some questions for novice English learners, and adjusting some for intermediate and more advanced English learners. Differentiated questioning gives all students the opportunity to participate in rich class discussions.

When an English learner student responds (and the answer moves the discussion forward), the teacher may use "mathematical rephrasing" to clarify the idea for all students and model desired academic discourse. For example, if an English learner says, "*y equal x square* ($y = x2$) *is* a curve," the teacher might respond, "Yes, the graph of *y equals x squared* is a parabola (drawing or pointing to it), a line that curves." The rephrasing is most helpful to the English learner if it is just a level above what the student produced independently. The student may choose to repeat the teacher's rephrased statement, but should not be asked to do so. Mathematical rephrasing helps all students gradually develop much more sophisticated academic discourse skills. This can happen in a safe, respectful environment because students feel comfortable with their classmates and recognize the importance of everyone's contributions to group learning. When English learners are involved in class discussions where many students repeat the important terms and ideas, they have a greater opportunity to comprehend concepts and thought processes. To support this kind of learning, a teacher might invite students to indicate their agreement by restating the teacher's or another student's statement.

In teacher-assisted instruction, before asking individual students to reply, the teacher may use Think-Pair-Share, to give students ample time to think and share answers with a partner before the whole-class discussion begins. This allows English learners to express their ideas comfortably with a partner before "going public" in front of the class. Think-Pair-Share is another way to build in repetition because it allows English learners to hear an important concept described in slightly different ways, first in pairs and then in a whole-class discussion.

Peer-Assisted Instruction

In peer-assisted instruction, small groups of students interact and learn as a team through collaborative or cooperative activities. Before students begin complex group activities, the teacher may need to model the expected group learning behaviors and establish rules of conduct. Students teach each other and learn together while the teacher monitors, guides, and models as necessary.

APPLICATIONS FOR ENGLISH LEARNERS

Some basic steps prepare students to work effectively in teams or small groups and ensure that English learners will be able to participate and learn. To design effective cooperative and collaborative activities the teacher makes sure that an activity is cognitively challenging for everyone, while varying the language demands students must meet in order to participate and contribute. The activity is also structured to be what Elizabeth Cohen[29] would describe as "group worthy," meaning that it necessitates collaboration and discussion.

By setting clear directions and expectations for group work, the teacher sets the best conditions for students to focus on learning. In classrooms with English learners, directions should be written as well as oral. When the teacher writes the directions before giving them orally, it provides an opportunity to check that they are clear.

Sometimes students working in groups misbehave or become passive because they do not understand the concepts or the task instructions. To remedy this, the teacher may need to initiate group work by modeling expected behaviors and gradually shifting ownership of the group learning process to the students. Assigning roles is one way to help groups manage their interactions and structure successful participation for everyone. For example, a novice English learner in a group could participate as the illustrator of key concepts, while more English proficient students are assigned to act as facilitator, writer, or reporter. As they gain more experience and success, students can choose their own roles or collaborate more interdependently. Regardless of how groups are structured, teachers should set the expectation and provide the opportunity for all students to learn and accomplish the goal of the lesson.[30]

Grouping decisions should serve the teacher's strategic goals. Grouping English learners by primary language; including an English learner in a group with more proficient English speakers; grouping students by characteristics other than English proficiency; or allowing students to choose groups by topic or interest may help some students feel more comfortable speaking in the group. In a classroom where a strong community has been established, students might sometimes be allowed to choose groups by friendship, as long as everyone understands that no student should feel unwanted. Often, teachers assign novice English learners to a group with a proficient English speaker so that the English learner hears language modeled and other students' rephrasing of what the teacher said.

The size of a group matters. Pairs and triads are more likely to keep all members involved. Foursomes or larger groups may provide more opportunity for diversity of ideas or be necessary because of the number of available resources (e.g., manipulatives), but one or more students may become marginalized or choose to be passive. Cooperative tasks in which each student is assigned a specific subtask can keep all students involved in a larger group such as a foursome. A teacher might start the year by guiding students to collaborate in pairs and then make the transition to triads and foursomes.

DIFFERENTIATING INSTRUCTION

Differentiating instruction[31] means using a variety of instructional strategies that target the diversity of students in the classroom — students with different learning styles, interests, special needs, and those who are also English learners. For English learners, differentiation means tailoring a specific strategy to fit their various levels of English proficiency. It does not mean creating an individualized lesson for each student. It means planning a variety of ways for students to interact with new concepts. It also means controlling speech and using word walls, visuals, and small-group learning activities to make input more comprehensible for English learners.

For example, a mathematics teacher who has English learners at two or three levels of proficiency (see chapter 3 for a description of English language development levels) may use the same teaching strategies for all students, but differentiate instruction by offering support that is tailored to students' levels of English language proficiency with a given strategy. When required to record their thinking, all students are encouraged and supported to draw pictures or write sentences. When students are expected to write several connected sentences, the teacher gives English learners templates with some portion of the text already completed, according to their language level. The most novice English learners receive a template that only requires filling in key words and phrases or drawing diagrams to express their thinking. English learners at a higher proficiency level are provided sentence starters and transition words between sentences to help them write connected ideas. (See chapter 5 for a more detailed discussion of such supportive strategies.)

Other ways the teacher can differentiate instruction for English learners include: Accompanying oral presentations with visuals to help students listen with greater comprehension; giving English learners note-taking outlines or sentence starters to help them capture key concepts in a challenging textbook; and providing hands-on activities to help English learners "see" and actively engage in learning mathematics concepts and procedures. When the teacher presents the big picture or main idea first, as a frame for the information that will follow, English learners are better prepared to concentrate on what is most important. It is important that direct instruction for English learners be delivered in small chunks, allowing them time to process the information. Connecting instruction to students' experiences and offering varied forms of support heighten all students' interest and personalizes instruction in a way that motivates students.

Some students learn better in small groups than they do individually. Small group talk gives English learners a chance for language repetition and practice, so differentiation also means planning for collaborative and cooperative learning activities. The focus of differentiation is to be aware of all the ways students are different from one another, and to plan to teach in ways that capitalize on those differences. The classroom techniques described in Figure 1.6 reflect a number of the ideas in this chapter about differentiating instruction for English learners.

FIGURE 1.6. Supporting English Communication for English Learners

The following classroom techniques have been found effective in supporting English communication and differentiating instruction for English learners:

» Tap into prior knowledge to give students richer context for what they will learn. At the same time, activating prior knowledge lets students anticipate vocabulary and terms they are likely to hear and enables them to use context to guess words they do not know.

» Provide wait time after asking a question — it may take English learners extra time to process back and forth in their primary language and English as well as to understand the question itself.

» Have students discuss with a partner or in small groups relevant information from prior mathematics lessons or personal experience; monitor group discussions; and then use a few examples to share with the class. Use flexible grouping in terms of primary languages spoken, English proficiency, general mathematics knowledge, friendships, and other criteria.

» Use multimodal presentations — visuals, word walls, graphic organizers, hands-on activities, etc. — during direct instruction and when summarizing or reviewing.

» Repeat and rephrase important concepts, keeping periods of lecture or reading brief and concise but highly contextualized. Present new words in the context of the lesson and apply words during the lesson, pausing to emphasize each key word.

» Use tiered lessons that address the same standards and topics but that adjust the language level to challenge without frustrating students. For example, plan opportunities to restate a chunk of oral instruction in simpler form for English learners, perhaps while other students do seatwork; provide texts at different reading levels; assign tasks that differ in language demands; assign learning activities to small groups in which more proficient English speakers rephrase concepts and English learners are assigned less language-demanding, but still mathematically rich, parts of the task.

During assessment, the teacher also differentiates. For example, if students are to write about what they have learned, they are not uniformly presented with a blank sheet of paper and the general direction to "Explain the Pythagorean Theorem and its application in the real world." It will be more appropriate to ask some English learners to respond orally to a series of guiding questions. Others may be provided with sentence starters or graphic organizers, or be asked to draw pictures with labels and write a few simple sentences that explain their pictures. Often the scaffolding strategies that are used to help English learners during instruction also are used to help them express what they have learned during assessment. Figure 1.7 presents an example of assessment differentiation that scaffolds English learners at different levels of proficiency.

In chapters 4 through 6, the ideas explored in this chapter about the three-phase model, modes of instruction, and differentiation of instruction will be made more concrete, as we introduce specific tools and scaffolding techniques that support English learners. Chapters 2 and 3 present information about language acquisition and expected skills at each language proficiency level, to provide

FIGURE 1.7. **Assessment That Accommodates Different Levels of English Proficiency**

In the example below, the teacher provides assessment accommodations at three levels of English proficiency. The goal is to learn as much as possible about what English learner students have or have not understood about mathematics content, not to demonstrate that they are not yet proficient in English.

Beginning. English learners are given a template and asked to visually represent key ideas in pictures, diagrams, or graphic organizers. Students include complete simple sentence starters and labels as appropriate to their level of language development. (See chapters 2 and 3 for descriptions of various levels of language development.)

A quadrilateral has 4 sides. Some quadrilaterals are parallelograms or have parallel sides (||), but some are not parallelograms. A trapezoid is a type of__________. A trapezoid has at least one pair of ______ sides. The shape of a roof of a house looks like a _________.

Intermediate. English learners complete a visual representation and also complete sentence frames that help them connect ideas.

A quadrilateral has 4 sides. If the sides are parallel (||), the quadrilateral is a _______. If the sides (of the quadrilateral) are not parallel, then… A trapezoid is an example of…

Source: Carr, J., Sexton, U., & Lagunoff, R. (2007). *Making Science Accessible to English Learners: A Guidebook for Teachers*. San Francisco, CA: WestEd.

mathematics teachers with a firm foundation for embarking on strategies to effectively teach English learners at different developmental levels. In chapter 2, we provide a brief overview of language acquisition theory as it applies to classroom practice; and we compare the cognitive demands of learning conversational English with those of learning more formal, academic English. In chapter 3, we describe students' language abilities at five levels of English proficiency using a simple chart, and list an array of corresponding teacher strategies.

ENDNOTES FOR CHAPTER 1

[1] Martin, T. S. (Ed.). (2007). *Mathematics teaching today*. (2nd ed.). Reston, VA: National Council of Teachers of Mathematics.

[2] National Research Council. (1999). *How people learn: Bridging research and practice*. Washington, DC: National Academies Press. See also NRC's *How students learn: History, mathematics, and science in the classroom* (2005).

[3] Chapin, S., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. In W. G. Martin, M. E. Strutchens, & P. C. Elliott (Eds.), *The learning of mathematics: Sixty-ninth yearbook* (pp. 113–128). Reston, VA: National Council of Teachers of Mathematics.

Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning*. Portsmouth, NH: Heinemann.

[4] Weiss, R.P. (2000, July). Brain-based learning: The wave of the brain. *Training & Development*, 20–24. Accessed February 6, 2006, from http://www.dushkin.com/text-data/articles/32638/body.pdf.

[5] Hawkins, B. (2005). Mathematics education for second language students in the mainstream classroom. In P. Richard-Amato & M.A. Snow (Eds.), *Academic success for English language learners*. New York: Longman.

Padron, Y.N. (1993). Teaching and learning risks associated with limited cognitive mastery in science and mathematics for limited English proficient students. *Proceedings of the third national research symposium on LEP student issues: Focus on middle and high school issues*. (Vol. II.) Washington, DC: U.S. Department of Education, Office of Bilingual Education and Minority Languages Affairs.

[6] Martin, T. S. (Ed.). (2007). *Mathematics teaching today.* (2nd ed.). Reston, VA: National Council of Teachers of Mathematics.

[7] National Research Council. (1999). *How people learn: Bridging research and practice.* Washington, DC: National Academies Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Hawkins, B. (2005). Mathematics education for second language students in the mainstream classroom. In Richard-Amato, P.A., & Snow, M.A. (Eds.), *Academic success for English language learners*, (p. 380). White Plains, NY: Pearson Education, Inc.

[8] National Council of Teachers of Mathematics. (2000). *Principles & standards for school mathematics*. Reston, VA: Author.

[9] Martin, T. S. (Ed.). (2007). *Mathematics teaching today.* (2nd ed.). Reston, VA: National Council of Teachers of Mathematics.

[10] Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning*. Portsmouth, NH: Heinemann.

Khisty, L.L., & Chval, K.B. (2002). Pedagogic discourse and equity in mathematics: When teachers' talk matters. *Mathematics Education Research Journal, 14*(3), 4–18.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

[11] Informal communication with a teacher in Idaho Falls, Idaho, November 21, 2008.

[12] Social learning can be defined as a group of students discussing ideas and helping each other to learn.

[13] Fillmore, L.W. (1976). *The second time around: Cognitive and social strategies in second language acquisition.* Unpublished doctoral dissertation, Stanford University.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

Gibbons, P. (2002). *Scaffolding language scaffolding learning: Teaching second language learners in the mainstream classroom*, (pp. 6–10). Portsmouth, NH: Heinemann.

[14] Anstrom, K. (1999). *Preparing secondary education teachers to work with English language learners: Mathematics* (NCBE Resource Collection Series, No. 14). Washington, DC: National Clearinghouse for Bilingual Education.

[15] Buchanan, K., & Helman, M. (1997). *Reforming mathematics instruction for ESL literacy students*. Washington, DC: ERIC Clearinghouse on Languages and Linguistics.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

Cummins, J., & Swain, M. (1986). *Bilingualism in education: Aspects of theory, research and practice*. London: Longman.

[16] Ball, L., & Stacey, K. (n.d.). New literacies for mathematics: a new view of solving problems. Accessed January 14, 2009, from http://extranet.edfac.unimelb.edu.au/DSME/CAS-CAT/publicationsCASCAT/2001Pubspdf/BallStaceyNewLits.pdf.

[17] Bielenberg, B., & Fillmore, L.W. (Dec. 2004–Jan. 2005). The English they need for the test. *Educational Leadership, 62*(4), 45–49.

[18] Hawkins, B. (2005). Mathematics education for second language students in the mainstream classroom. In P.A. Richard-Amato, & M.A. Snow, (Eds.), *Academic success for English language learners*(p. 380). White Plains, NY: Pearson Education, Inc.

Bay-Williams, J.M., & Herrera, S. (2007). Is "just good teaching" enough to support the learning of English language learners? Insights from sociocultural learning theory. In W.G. Martin, M. E. Strutchens, & P.C. Elliott (Eds.), *The learning of mathematics: Sixty-ninth yearbook* (pp. 43–63). Reston, VA: National Council of Teachers of Mathematics.

Freeman, D.J. (2004). Teaching in the context of English-language learners: What we need to know. In M. Sadowski (Ed.), *Teaching immigrant and second-language students: Strategies for success* (pp. 7–20). Cambridge, MA: Harvard Education Press.

[19] National Research Council. (1999). *How people learn: Bridging research and practice.* Washington, DC: National Academies Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Hawkins, B. (2005). Mathematics education for second language students in the mainstream classroom. In P.A. Richard-Amato, & M.A. Snow, (Eds.), *Academic success for English language learners*, (p. 380). White Plains, NY: Pearson Education, Inc.

[20] For example, mathematics curricula that use a similar structure are CME Project, Aim for Algebra, Connected Mathematics, IMP, Math Thematics, and UCSMP.

[21] The definitions of the three phases are loosely adapted from *Strategic Science Teaching for Grades K–12* (2002), a framework developed by the California County Superintendents Educational Services Association (CCSESA) Curriculum and Instruction Steering Committee (CISC) Science Subcommittee.

[22] Vang, C. (2004). Teaching science to English learners. *Language Magazine, 4*(4). Adapted with permission of *Language Magazine*, http://www.languagemagazine.com.

[23] Echevarria, J., Vogt, M., & Short, D.J. (2008). *Making content comprehensible for English learners: The SIOP® model* (3rd ed., pp. 131–132). Boston: Allyn & Bacon.

[24] Different states use different terms such as English Language Development (ELD), English Language Proficiency (ELP), or English as a Second Language (ESL) Standards.

[25] Bybee, R.W. (1997). *Achieving scientific literacy: From purposes to practices.* Portsmouth, NH: Heinemann.

[26] Krashen, S.D. (1985). *The input hypothesis: Issues and implications*. New York: Longman.

Krashen, S.D. (1981). *Second language acquisition and second language learning*. New York: Pergamon.

Long, M.H. (1981). Input, interaction, and second language acquisition. In H. Winitz (Ed.), *Native language and foreign language acquisition: Annals of the New York Academy of Science* (379, pp. 259–278).

[27] Wormeli, R. (2005). *Summarization in any subject* (p. 5). Alexandria, VA: Association for Supervision and Curriculum Development.

[28] Black, P., & William, D. (1998). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*(2), 139–149. Accessed February 6, 2006, from http://www.pdkintl.org/kappan/kbla9810.htm.

[29] Cohen, E.G. (1994). *Designing groupwork: Strategies for the heterogeneous classroom*. New York: Teachers College Press.

[30] Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning.* Portsmouth, NH: Heinemann.

[31] A number of sources inform this discussion of differentiating instruction. For example:

Cole, R.W. (Ed.). (1995). *Educating everybody's children: Diverse teaching strategies for diverse learners*; and Cole, R.W. (Ed.) (2001). *More strategies for educating everybody's children.* Alexandria, VA: Association for Supervision and Curriculum Development.

Gregory, G., & Chapman, C. (2001). *Differentiated instructional strategies: One size doesn't fit all.* Thousand Oaks, CA: Corwin Press.

Silver, H.F., Strong, R.W., & Perini, M.J. (2000). *So each may learn*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A. (1999). *The differentiated classroom*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A., & McTighe, J. (2006). *Integrating and differentiating instruction: Understanding by design.* Alexandria, VA: Association for Supervision and Curriculum Development.