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Making the Most of Mistakes

Math Pathways & Pitfalls Improves Student Learning

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Learn from mistakes. That's good advice, whether coming from business gurus or seasoned parents. But it is advice rarely applied in the typical elementary mathematics lesson, where there is so much pressure to get the "right" answer that teachers usually avoid dwelling on students' errors. And according to research behind WestEd's Math Pathways & Pitfalls (MPP) initiative, this typical approach misses important opportunities for learning.

One of the most prevalent instructional approaches in U.S. mathematics classes, as documented by the Trends in International Mathematics and Science Study (TIMSS), consists of the teacher showing students how to find solutions to a few examples on the board. Students then work on their own to find solutions to similar examples. In this approach, teachers intentionally avoid common errors and misconceptions because they're concerned about causing students to remember the wrong solution process, notes Carne Barnett-Clarke, former Senior Research Associate at WestEd and MPP principal investigator.

However, students' errors and misconceptions often persist. Even after a complete unit on decimals, for example, teachers may be surprised to find that many of their students still say that 0.7 + 0.5 = 0.12 (correct answer is 1.2).

Barnett-Clarke says such errors may be symptomatic of more significant weaknesses: perhaps an inability to recognize when a solution is unreasonable, or a failure to make sense of mathematical symbols.

In response to students' mistakes, most teachers show them more examples of correct solutions and then give the students more practice. But this approach does nothing to address the students' underlying conceptual understanding. Students may continue to look only at the surface features of the problems. They see a 7 and a 5 and a plus sign, for example, so they add the numbers to get 12, then put a decimal point in front.

Without any deeper understanding of the mathematical concepts and of the symbols used to represent the concepts, students will continue to think an incorrect solution makes perfect sense, even when it is pointed out as an error.

Mathematical Concepts, Misconceptions, and Language

Research on MPP, a long-term WestEd initiative, has found that teachers can modify how they usually teach mathematics and significantly boost students' achievement. MPP provides teacher professional development and a series of classroom lessons for grades K-7 to teach students how

to participate in high-level, productive discourse about complex mathematical concepts, including focusing explicitly on errors.

As students begin recognizing common pitfalls and misconceptions that can derail their reasoning, they are more likely to ask themselves, "Where can my thinking go wrong?" says Barnett-Clarke. "That means they are less likely to develop stubborn misconceptions or just fall into mistakes, especially when taking a test."

Barnett-Clarke says MPP shows students how to use the high-level, sophisticated, precise kind of "academic language" they need to talk about mathematics in a meaningful way. "It goes beyond just learning math vocabulary and symbols to being able to explain one's reasoning using mathematical syntax," she says. "It's knowing how to use 'if . . . then' statements, make conjectures and justifications, and offer arguments that support a way of solving a problem."

Benefits for English Learners

While MPP's instructional practices have been found helpful to all students, its emphasis on the support of verbal and symbolic language is proving especially effective when it comes to boosting the math achievement of English learners.

Scaffolding mathematical discussions is a particularly important way to provide effective instruction for language minority students, says Alma Ramirez, WestEd Senior Research Associate who co-directs MPP with Barnett-Clarke. MPP lessons provide scaffolding through "attention to vocabulary, support for students' writing, the use of various modes of communication, and support for participation in academic discourse in mathematics." Each lesson, for example, begins with the introduction of important mathematical vocabulary and includes language acquisition strategies.

MPP lessons give students practice presenting complete and coherent explanations of math problems, and help them become comfortable expanding upon, proving, or disproving mathematical ideas. Barnett-Clarke says that students who master these kinds of discussion skills "won't be thrown when they get to high school or college by a language that really sounds different to them."

Teachers can use MPP "Discussion Builders" posters to give students examples of how to build on, analyze, and justify their ideas or the ideas of their classmates.

For example, prompts on the posters encourage students to present alternative ideas by stating, "I wonder what would happen if . . ." They can expand on a classmate's ideas with a statement that begins, "I'd like to add to _____'s idea." Or they can pose additional questions by asking, "Would that be true if . . . ?"

Barnett-Clarke says this kind of attention to language is rare, especially in math classes. Most people, including many teachers, erroneously think of math as language-free. "But elevating mathematical discourse by using academic language is a powerful way to gain information and understanding."

"This approach is particularly important," says Ramirez, "because much of the schooling for language minority students has excluded them from the academic discourse community of the classroom." Too often, she says, these students have been relegated to "watered-down, remedial" math lessons that included little talk, few representations, and minimal reading or writing about mathematics.

MPP, by contrast, provides language support aimed at helping all students develop a "robust understanding" of mathematical concepts. Teaching guides, for example, provide mathematically rigorous examples and explicit discussion probes so teachers learn how to guide students toward increasingly sophisticated levels of mathematical understanding.

Proven Impact

MPP has been funded by grants from the U.S. Department of Education, the National Science Foundation (NSF), and the Stuart Foundation. As part of the NSF grant, WestEd researchers conducted a randomized experimental study of MPP involving 100 classrooms. MPP was shown to have a "significant impact on the mathematics achievement of a diverse population of students," reports Barnett-Clarke. And, according to the Coalition for Evidence-Based Policy, MPP has "produced meaningful effects on educational outcomes."

Barnett-Clarke says some of the most gratifying preliminary findings to come out of recent MPP classroom observation studies suggest that both teachers and students may be using MPP instructional techniques during non-MPP lessons.

"The preliminary feedback we're getting convinces us that teachers are adapting a lot of these instructional techniques to other lessons," she says. "They tell us it's not just the way they teach math that's changing, but rather the way they teach overall."

And as for the students, Barnett-Clarke reports that it's not unusual, even during non-MPP lessons, to hear them say things like, "That's a pitfall," and then go on to explain why it is a pitfall.

"What occurs through an MPP lesson just naturally influences teaching and learning," says Barnett-Clarke. "There's no need to make lots of adaptations." She describes the approach as "transformative to the students and their teachers."

MPP posters, which come with a teaching guide, are available from WestEd. Barnett-Clarke expects the entire program, which also includes videos for teachers and students, to be published early in 2008.

For more information, visit Math Pathways & Pitfalls at http://www.wested.org/cs/mpp/print/docs/mpp/home.html.