Conquering the Pitfalls on the Pathway to Math Success

Claudia, a sixth grader in a bilingual education class, "hated" math at the beginning of the school year. And she was very shy about speaking in class. As the year progressed, however, her teacher, Tatiana Peugnet, observed that "Claudia gradually became more confident about speaking out and sharing her math work."

Part of the reason for that transformation, Peugnet believes, is the approach to mathematics learning that is central to WestEd's Math Pathways & Pitfalls curriculum, which Peugnet uses in her classroom. A K–8 intervention curriculum that can be integrated into the core curriculum or used in supplemental programs, Math Pathways & Pitfalls develops students’ mathematical language and analytic skills in order to foster a deep understanding of key mathematical concepts.

Over time, Peugnet found that the curriculum’s approach to learning helped her develop a safe, productive classroom atmosphere for academic discussions. "Initially, one of Claudia's big fears was having the class disagree with her answers, but she really began taking more risks as the year went on. Math actually became one of the subjects she really enjoys."

Research—including two national randomized trial studies—shows that even fairly limited exposure to the Math Pathways & Pitfalls curriculum significantly improves student achievement. A five-year study (2005–10) funded by the U.S. Department of Education’s Institute of Education Sciences (IES) found that a total of 30 hours of exposure to Math Pathways & Pitfalls lessons over two years raised both standardized and project-administered mathematics test scores for the full sample of students, of which approximately 70 percent were Latino, 55 percent were English language learners, and 75 percent were eligible for free or reduced-price meals.

Alma Ramirez, Co-Director of Math Pathways & Pitfalls, says that the IES study results were particularly notable because few studies have been able to report successful interventions for raising the mathematics achievement of English language learner and Latino student groups.

"The lesson structure was intentionally designed to create a safe way for students, particularly ELL students, to share their mathematical processes and reasoning," Ramirez says. "We’ve built in supports along the way so it’s safe for students, and even teachers, to take intellectual risks."

Going Beyond Right and Wrong Answers

Each Math Pathways & Pitfalls lesson takes the unique approach of confronting misconceptions head-on by asking students to analyze both correct and flawed ways of solving a problem. In order
to build a sound conceptual understanding of the topic, students are first shown correct solutions and their justifications. Students are then presented examples of "pitfalls," which address some of the most common misconceptions cited in the research literature on student thinking—ones that are often used as distractors on state and national assessments. Ramirez notes that the goal for presenting students with both correct and incorrect solutions isn't for students to guess which one is right, but to enable them to understand and discuss why the correct answer works and why the reasoning behind the incorrect answer is flawed.

The curriculum encourages students to explicitly examine and explain their mathematics thinking and processes. This metacognitive approach is particularly important, notes Ramirez, because when students misapply a mathematical procedure it's generally because they don't understand the concept that underlies the procedure. By getting into the habit of reviewing their thinking and asking themselves whether or not a solution makes sense, students also become more conscious of how they learn and how they can monitor their own learning and problem solving.

"We all learn from our mistakes," says Leanna Baker, a first-grade teacher in Hayward Unified School District in Hayward, California, who regularly uses Math Pathways & Pitfalls strategies in her classroom. "Math Pathways & Pitfalls has helped me build a classroom culture where it's safe for students to talk about the errors they've made—and to discuss the reasoning that led to the errors and how to avoid that in the future. It's really helped my students learn to explain their thinking and take responsibility for their own learning."

**Digging Deep Into Important Ideas**

As states and districts begin to implement the Common Core State Standards, teachers are finding that Math Pathways & Pitfalls aligns well with the math standards' increased focus on students' ability to demonstrate conceptual understanding and provide explicit justifications for their approaches to solving a problem. For instance, a Math Pathways & Pitfalls fourth-grade lesson on adding fractions goes beyond asking students to focus solely on getting the right answer to a numerical equation, such as $\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$. Instead, the lesson guides students to understand and articulate the mathematical ideas behind the procedure, such as what makes fractions equivalent and why a common denominator is required to add fractions. Instruction employs strategies such as having students visualize the addition of fractions along a number line and other types of modeling.

"The Common Core is asking for student reasoning," says Peugnet, "which is something Math Pathways & Pitfalls helps scaffold by prompting students to justify their answers and analyze common misconceptions. Rather than giving students 25 problems at a time to plow through—which provides practice in the mathematical procedures—Math Pathways & Pitfalls encourages deeper discussion of fewer problems. With this curriculum, my students learn the concepts underneath the procedures and how to talk about the problems they need to solve."

To enable students to discuss mathematics concepts, the curriculum focuses explicitly on building students' discipline-specific academic language. The goal is to help students become comfortable with the unique ways mathematics language is structured, through such practices as constructing "if...then" scenarios or posing a conjecture about how to solve a problem or a proof of a solution. Each lesson opens by introducing a few strategically selected key mathematics terms; for
instance, the lesson focused on adding fractions begins with students learning how to use the terms "addends," "common denominator," "unlike," and "sum" when discussing the mathematical reasoning behind each problem.

Also, embedded throughout the curriculum’s teaching guides are terminology and guidance to help teachers make the curriculum more accessible for English language learners. For example, in the fractions lesson, the curriculum reminds teachers to: "Note [to students] that we don’t read halves as ‘twos,’ thirds as ‘threes,’ or fourths as ‘fours.’ The different endings for the denominators of a fraction denote how many equal parts the whole is divided into."

Another element of Math Pathways & Pitfalls that helps foster metacognitive discussion of mathematical concepts is the curriculum’s "Discussion Builders," which prompt students to learn through active participation. Integrated into each lesson, the Discussion Builders provide built-in scaffolding to foster increasingly sophisticated use of academic language and reasoning as students progress through the grades. As opposed to sentence stems for discussion that only allow room for closed, finite answers, the Discussion Builders are designed as open-ended prompts that encourage students to think about and question the concepts they’re learning, says Ramirez.

"The curriculum’s focus on language development is my favorite aspect of Math Pathways & Pitfalls," says teacher Leanna Baker. "Math vocabulary is unique—it’s not something you’re born with or your parents teach you. The curriculum gives students the opportunity to build both the conceptual and linguistic understanding of mathematics."

In addition to the IES study, a large-scale experimental study funded by the National Science Foundation found that using just 15 hours of Math Pathways & Pitfalls lessons in place of regular mathematics lessons boosted student achievement during the course of one school year. Ramirez thinks one of the reasons that even a limited amount of exposure to Math Pathways & Pitfalls can produce such dramatic results is that the approach and strategies translate beyond just the individual lessons.

"We’ve observed that using Math Pathways & Pitfalls curriculum helps teachers and students acquire ‘metacognitive habits of mind’ that they can apply to all academic work," says Ramirez. "Our curriculum is only a supplemental or intervention curriculum. The point isn’t to fill a whole school year of lessons—the curriculum is designed to build a strong conceptual understanding and a way to think and talk about mathematics more deeply. This learning, then, translates to other mathematics lessons."

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