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Learning Exponential Functions With Immersive Virtual Reality

Outcomes of a Randomized Controlled Trial as Part of the Prisms NSF SBIR Phase II Project

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Study Overview

WestEd conducted a randomized control trial (RCT) study for Prisms immersive virtual reality (IVR) in September 2022–April 2023 to test the hypothesis that classroom implementation of the Prisms IVR platform positively impacts students' engagement and algebra (exponential functions) proficiencies. WestEd, an education research agency, conducted evaluation activities and testing of the Prisms curriculum and experience under the Prisms NSF SBIR Phase II grant.

Research Questions

The following primary research questions guided this study.

Student Questions

1. Does the use of Prisms impact students' engagement, perseverance, and mindset toward math learning?
2. Do students who use Prisms perform better on an exponential functions–focused algebra test than their counterparts?
3. Do students' perceptions of themselves as math learners change after the using Prisms?

Teacher Questions

1. Do teachers report any changes in student engagement, perseverance, and mindset toward math learning after the use of Prisms, as compared to the business-as-usual comparison group?
2. Do participating Prisms teachers implement Prisms modules as intended by the developer? To what extent does the implementation and fidelity vary?

Summary of Findings

- Students in the Prisms treatment group outperformed students in the business-as-usual control group on a researcher-developed measure of exponential functions.
- Student self-reports of mathematical mindset and mathematical confidence did not differ between treatment and control students.
- Teachers reported that student engagement increased with the use of Prisms lessons, particularly with regard to their participation in whole-class discussions.
- Future work should expand beyond exponential functions and investigate the relative impact of additional Prisms modules on students' mathematical learning.

About Prisms

Prisms is a supplemental curricular resource for mathematics and science teachers. This study is focused on resources for an Algebra 1 course—specifically the module designed to introduce exponential functions. The modules are intended to introduce a mathematics topic and provide a motivating context to continue studying the mathematics idea. Prisms uses immersive virtual reality to engage students in problem-driven, tactile, and visual learning. Students use head-mounted displays (e.g., Oculus or Pico) to enter into a virtual world, and through the embodiment of an avatar, students experience a phenomenon that poses a real-world problem that needs to be addressed. Next, students mathematize the phenomenon in a virtual laboratory as they manipulate mathematical objects and begin to develop mathematical understandings that support making further sense of the phenomenon and possible solutions. Each Prisms module is focused on one particular phenomenon that is explored for the purposes of developing particular mathematical understandings and skills. For instance, in the exponential functions lesson that is the focus of this study, students experience and mathematize a viral outbreak. Figures 1 and 2 provide example screenshots for what students see and interact with in the IVR environment. For more information about Prisms, visit www.prismsvr.com.

The Structure of a Prisms Module

Each Prisms module contains three parts and is designed to be implemented over 3 days. Each module follows a similar structure. Students use the IVR during the first 2 days, and the 3rd day is a synthesis lesson that involves a paper worksheet and class discussion.

IVR Day 1

IVR Day 1 primarily provides students with a visceral experience of a phenomenon (e.g., melting glaciers, viral spread). The module teacher guides suggest that IVR Day 1 be implemented in three phases: First, the teacher frames the upcoming experience by posing an activation question (approximately 10 minutes). For instance, in the exponential functions module teachers might ask: “What was it like at the beginning of the pandemic when the virus first began to spread? What do you know about how viruses spread?” Second, students are engaged in IVR (approximately 20–25 minutes). While in the IVR, students first visit a location that provides the context for the upcoming problem (e.g., glacier, air traffic control tower, food court). Then students are taken to the virtual laboratory where they accept a mission, such as "How many weeks until the hospitals in your community are going to reach capacity?" To solve this mission, they begin to abstract from the physical experience of a virus spreading in a food hall using simulations and tactile interactions (e.g., mathematical representations) that reflect the structure or phenomena they observed in the earlier virtual experience. Finally, the module teacher guides suggest a debrief conversation (approximately 15 minutes) that is aimed at recording salient aspects from the IVR experience that will be used in the subsequent lessons to develop formal mathematical representations.

IVR Day 2

In the second portion of the IVR, students are in the laboratory exploring, manipulating, and formalizing mathematical representations from IVR Day 1. Similar to IVR Day 1, the module teacher guides suggest three phases: First, the teacher frames the upcoming laboratory session by using **activation questions** (approximately 10 minutes) to recall important features from IVR Day 1 and frame the upcoming session. For instance, in the exponential functions module, the guide suggests that teachers can frame the experience by stating the following:

> *In the first part of this module, you physically experienced the spread of a virus, joined a task force, and investigated the impact of city structure and culture on viral spread. Now, you'll continue your efforts towards accomplishing your mission: to determine the number of weeks until our hospitals reach capacity.*

Second, students enter the **virtual lab** (approximately 20 minutes). On this second day, students are presented with mathematical representations, such as tables and graphs, to manipulate to express various aspects of the earlier explored phenomenon. Finally, the module teacher guides suggests a **debrief** conversation (approximately 15 minutes) that is focused on developing formal notation for the mathematics explored in the virtual laboratory. In the case of exponential functions, the debrief conversation is aimed at the contextualized meanings of the terms in an exponential function expressed as $y = a(b^x)$, as well as how the function representation as an equation relates to numerical values in a table.

Figure 1. Screenshots From IVR Day 1 for the Linear Functions Module

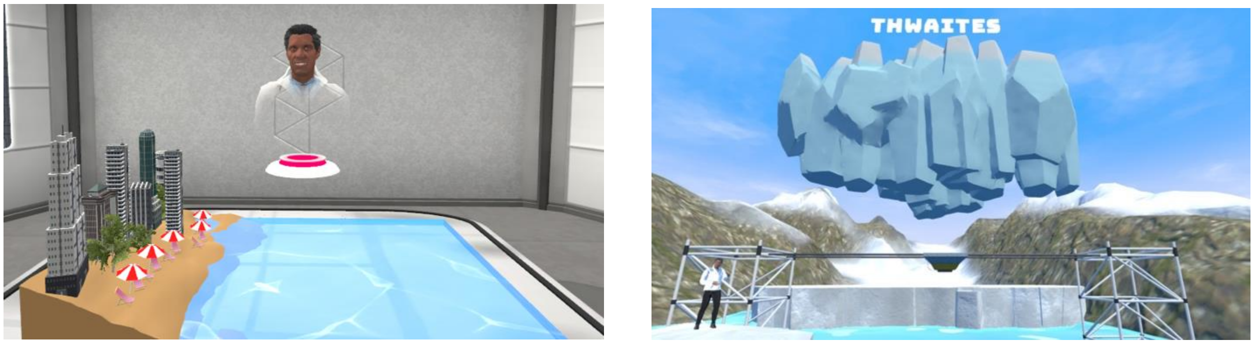
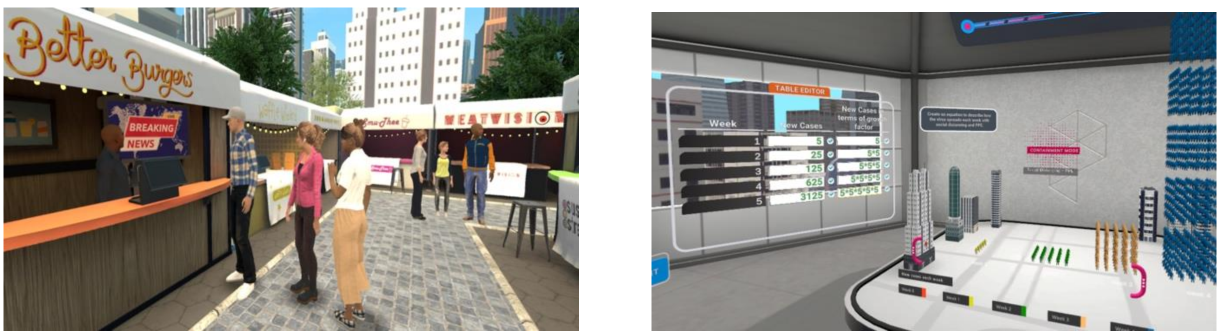


Figure 2. Screenshots From IVR Day 1 and Day 2 for the Exponential Functions Module



Synthesis Lesson

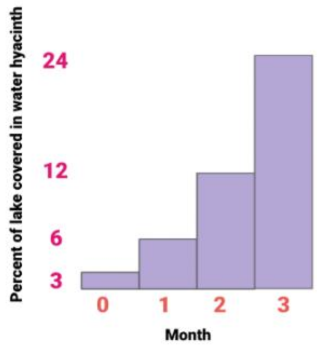
The third and final portion of a Prisms module consists of the synthesis lesson. The synthesis lesson relies on a paper worksheet and is intended to formalize the mathematical ideas introduced during IVR Day 1 and Day 2. For instance, in the exponential functions module, students identify the growth factor and initial values in various situations represented by exponential growth in tables and graphs to write the exponential function in the form $y = a(b^x)$. The module teacher guides suggest six phases of a synthesis lesson: First, teachers **frame** (2 minutes) the activity by summarizing the mathematics used in the previous days (e.g., "[You created] graphs, tables, and equations that modeled the spread of the virus week to week. These are called exponential models."). Second, the teacher directs students through a **launch problem** (approximately 10 minutes) to recall and apply mathematical ideas from the previous 2 days using a similar scenario. Next, in the **work time problem** (approximately 7 minutes) students work independently or in small groups to practice using the newly developed concepts. Then, the teacher facilitates a whole-class mathematics discussion, named **discourse** (approximately 10 minutes). In the discourse portion of the lesson, the teacher chooses various student work to share and discuss. The module teacher guides provides several discussion prompts. For instance, in the exponential functions module, teachers may choose from the following questions or generate their own:

> How do the table, graph, and equation all describe the number of new infections as a function of time?
>
> When the number of new infections reaches 5,000 per week, the city plans to start closing some public spaces. The City Council is worried that this will happen by Week 8 of the pandemic. Use your equation to find out if they're correct. Explain how you figured it out!
>
> In the IVR module, we saw that in City C, the viral spread had a growth factor of 5. Why is the growth factor in the problem we just solved less than that? What evidence do we see in our table, graph, and equation?

Students then work on a problem set during **independent practice** (20 minutes; see Figure 3). Prisms provides three problems sets, with varying levels of scaffolding, to differentiate based on students' current level of understanding. All independent practice problems ask students to apply their learning to a new context. In the exponential functions module, the new context is a pond with an invasive water hyacinth plant growing in it. Finally, the **closing debrief** (5 minutes; see Figure 4) is a whole-class activity where students respond to a representation to find an error, identify salient features, or engage with in other ways to solidify their mathematical understanding.

Figure 3. Example of a Portion of a Scaffolded Independent Practice Problem Provided by Prisms to Be Used During the Synthesis Lesson

Water hyacinth is a very pretty but very invasive plant species that spreads quickly over bodies of water, making it difficult to engage in water-based activities such as boating, fishing, and swimming. Lake Minerva is a small resort town that relies on summer tourism to support the community, but the spread of water hyacinth makes it very difficult to enjoy activities in the lake. Local scientists are studying how quickly water hyacinth is spreading over the course of winter and spring so they can help the town prepare for the upcoming tourist season. The diagram illustrates their findings.



a. Use the diagram to fill in the second column in the table to record the percent of Lake Minerva covered in water hyacinth as it spreads each month.

Month	% of Lake covered in water hyacinth
0	3
1	
2	
3	

b. What pattern do you see between the outputs (% of lake covered in water hyacinth) in your table? Describe this pattern with a sentence or two or annotate the table.

This image is fully described in Appendix C.

Figure 4. Example of a Closing Debrief Problem Provided in the Teachers' Guides as a Part of the Synthesis Lesson

Closing Debrief (5 mins.)

Today we saw how different exponential models can help us predict what will happen in the future in situations with a constant growth factor. Let's make sure we really understand the difference between exponential and other types of functions before we leave today:

A student in another homeroom filled in this table to describe the viral spread in City D. Remember that we said in Week 0 there was 1 new case, and that each week each person spread it to 3 more people.

Week	Number of New Cases
0	1
1	3
2	6
3	9
4	12

Where is the error in the student's reasoning? What might this graph look like? How is this graph different from an exponential graph?

Give students a few moments to discuss or jot a response to this question, working alone or in partners. Share exemplar responses (that the student added 3 each week instead of multiplied; the graph would be linear).

This image is fully described in Appendix D.

Methods

In this section we describe the study intervention and measured use to study the intervention.

Description of the Study Intervention

This study was designed as a randomized control trial (RCT). Algebra 1 teachers in middle school and high schools were randomly assigned into two groups: a business-as-usual (BAU) control group and a treatment group. The BAU teachers were required to teach through their linear functions and exponential functions units by the end of study period. Teachers assigned to the treatment group were asked to implement and incorporate two Prisms virtual reality module(s) into their existing mathematics instruction; a third module was optional. The three modules were:

- linear functions,
- exponential functions, and
- systems of linear functions (optional).

All modules engage students in real-world applications of mathematics. The linear functions module explores the melting rate of glaciers and related rising of sea levels. The exponential functions unit explores the spread and containment of a virus, and the systems of linear functions unit explores flight paths from the perspective of air traffic controls. These contexts are intended to motivate students' interest in mathematics, and thus the modules are designed to be used as an introduction to the unit. Each module consists of 3 days of activities. In Days 1 and 2, students explore the context in a IVR environment, and on the 3rd day, the mathematical ideas are solidified with a synthesis lesson that is supported with a paper worksheet and discussion. Teachers are supported to enact the lessons with a lesson guide that provides teachers with anticipated timing for various aspects of the lessons, suggested "look fors" to monitor students' understanding, and suggested prompts to facilitated discussion.

Participants

Of the 22 high school Algebra 1 teachers from Ohio participating in the RCT, only 21 teachers were included in the analytic sample due to student-level attrition. None of the participating study teachers had participated in any previous Prisms studies. The student-level assessment and survey collected responses from 514 middle and high school Algebra I students. Tables 1–7 provide a summary of school and student participant information for treatment and control conditions.

Table 1. Prisms RCT Analytic Sample—Count of Gender

	Boy or man	Girl or woman	Nonbinary, other, decline to state	Grand total
Control	125	108	19	252
Treatment	110	121	31	262
Grand total	**235**	**229**	**50**	**514**

Table 2. Prisms RCT Analytic Sample—Count of Race/Ethnic Background

	American Indian or Alaska Native	Asian or Asian American	Black or African American	Hispanic or Latino	Native Hawaiian or other Pacific Islander	Prefer not to answer	Two or more races	White or Caucasian	Grand total
Control	4	32	17	11	2	14	14	158	252
Treatment	1	17	6	8	2	20	14	194	262
Grand total	**5**	**49**	**23**	**19**	**4**	**34**	**28**	**352**	**514**

Table 3. Prisms RCT Analytic Sample—Count of Student School Locale Information

	Rural	Urban*	Grand total
Control	45	207	252
Treatment	118	144	262
Grand total	**163**	**351**	**514**

*** Urban includes Suburban and City**

Table 4. Prisms RCT Analytic Sample—Count of Students in Title 1 Versus Non–Title 1 Schools

	Title 1	Non-Title 1	Grand total
Control	156	96	252
Treatment	153	109	262
Grand total	**309**	**205**	**514**

Measures

In this section we describe the measured used to study the intervention. This includes the student surveys, exponential functions assessment, classroom observations, and teacher interviews.

Student Pre- and Post-Attitudinal Survey

At the beginning of the school year, all students (in treatment and control classrooms) completed an attitudinal survey. The survey consists of two subscales: the Mathematical Mindset subscale, focused on mathematical engagement, motivation, and enjoyment and the Confidence with Mathematics subscale, which focused on math self-efficacy. Students took the same survey once they had completed the exponential functions unit, which marked the end of the study period for that classroom. Students in treatment classrooms who experienced the Prisms IVR were also asked to rate their agreement regarding the extent to which the IVR experience supported their learning. See Appendix A for example survey items.

Student Pre- and Post-Exponential Functions Assessment

Prior to beginning the exponential functions unit, all students (in treatment and control classrooms) completed a 27-question researcher-designed assessment of their knowledge, understanding, and skills related to exponential functions. In this assessment, students were asked to recognize and differentiate real-world contexts related to linear, exponential, and other functions; interpret within a context the meaning of values within a given exponential function; and write exponential functions given a context. Because the Prisms IVR lessons are intended to be used at the beginning of the unit and motivate students' interest throughout a unit of instruction of exponential functions, the assessment included items that were within the scope of the Prisms module, as well as assessment items that are outside the scope of the Prisms modules but are typical goals within an exponential unit. For instance, the assessment included items related to exponential decay and exponential growth rates. Students took the same assessment once they had completed the exponential functions unit, which marked the end of the study period for that classroom.

Remote Classroom Observation

WestEd researchers completed remote classroom observation via Zoom. For all treatment teachers, we attempted to observe 1 day during the implementation of the IVR activities and 1 day of the synthesis lesson. When remote classroom observations were not feasible, WestEd researchers conducted short lesson debriefs after teachers completed their Prisms modules to gather information on how the day's lesson proceeded.

Teacher Interview

After the implementation of the exponential functions unit, five control teachers and five treatment teachers were selected to participate in 45-minute Zoom interviews with a WestEd researcher. Control teachers answered questions about their approach to mathematics instruction, generally, and their teaching of the exponential functions, specifically. Treatment teachers answered questions about their integration of Prisms into their regular instructional practices, their approach to teaching exponential functions, and their perceptions of how Prisms supported students' mathematics learning and engagement.

Table 5. Treatment Condition Participant Summary

Teachers	Number of students included in analytic sample
Teacher A	12
Teacher B	17
Teacher C	11
Teacher D	37
Teacher E	38
Teacher F	12
Teacher G	26
Teacher H	17
Teacher I	70
Teacher J	22
Total	**262**

Table 6. Control Condition Participant Summary

Teachers	Number of students included in analytic sample
Teacher K	18
Teacher L	20
Teacher M	7
Teacher N	58
Teacher O	15
Teacher P	31
Teacher Q	27
Teacher R	5
Teacher S	21
Teacher T	17
Teacher U	33
Total	**252**

Table 7. Participant Summary

Student demographics	Treatment	Control
Gender		
Female	42%	50%
Male	46%	43%
Nonbinary	3%	3%
Other	2%	2%
Prefer not to answer	6%	6%
Race/ethnic background		
American Indian or Alaska Native	0%	2%
Asian or Asian American	6%	13%
Black or African American	2%	7%
Hispanic or Latino	3%	4%
Native Hawaiian or other Pacific Islander	1%	1%
Prefer not to answer	8%	6%
Two or more races	5%	6%
White or Caucasian	74%	63%

Findings

In this section we report findings related to our research questions. First, we present the baseline comparisons of how students and teachers described classroom practices in the surveys previously described. Then we report findings related to students' mathematics achievement and engagement. Finally, we describe classroom implementation.

Baseline Comparison of Teacher Practices

As described in the methods above, students and teachers completed surveys to describe how their classes engaged in mathematical work. In this section, we report findings from the analysis of those surveys.

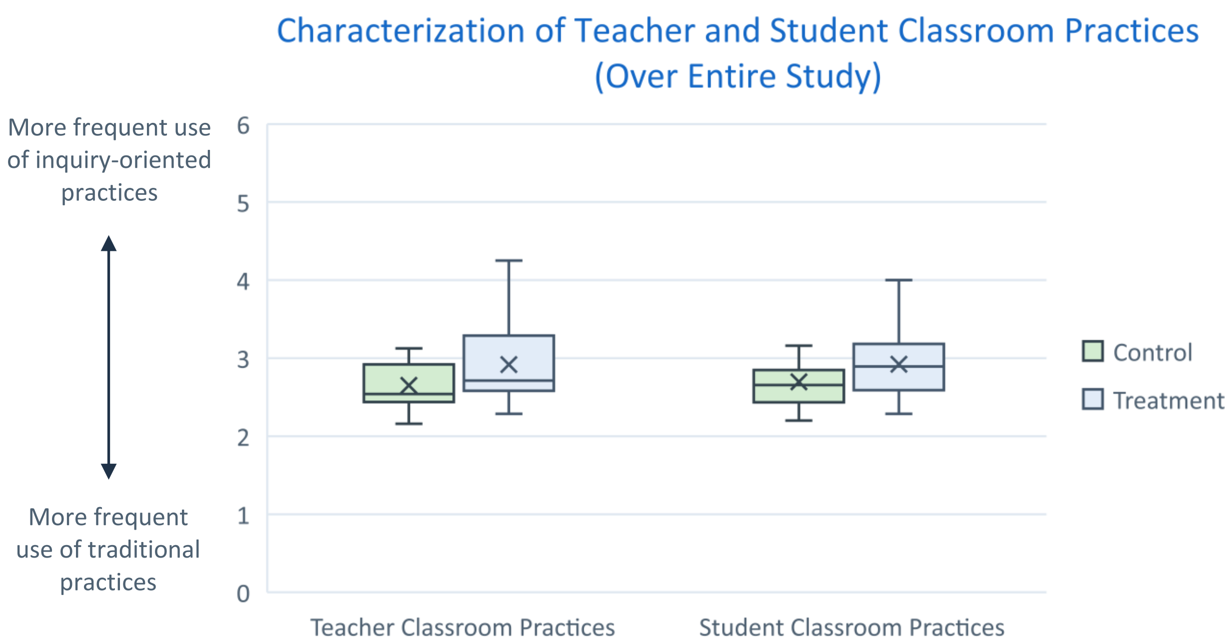
Treatment and Control Teachers Reported Similar Teaching Practices

Teachers were asked to complete a weekly log to indicate the primary mathematics topic in their class for the prior week. If teachers selected that they were teaching a topic for which they were also to complete a Prisms module (i.e., linear functions, exponential functions, or systems of linear functions) teachers answered questions about their teaching practices. We received 70 and 89 teacher logs from treatment and control teachers, respectively. Treatment teachers, on average, submitted 7 logs over the course of the study (with a range of 2 to 12 submitted per teacher). Control teachers, on average, submitted 8 logs over the course of the study (with a range of 4 to 10 submitted per teacher).

Figure 5 shows the distribution of each teacher's average as reported on the teacher logs. In general, a teacher with a higher average uses inquiry-oriented teaching practices more frequently than a teacher with a lower average. Scores close to zero are indicative of teachers using traditional teaching practices. Teacher classroom practices refer to the teacher's instructional moves. Student classroom practices refer to the nature of students' opportunities for learning (as reported by the teacher). See Appendix B for items asked on the teacher log. Overall, teachers reported similar teaching practices over the course of the study. The treatment teachers indicated slightly more inquiry-oriented practices, but 75 percent of the treatment teachers were within the same range of the control teachers' scores. Most importantly, the treatment and control group teachers did not report drastically different teaching practices than those in the other group.

The distribution of each teacher's average for the type of classroom practices students engaged in was similar and consistent with what was reported about classroom practices. Once again, the higher score is indicative of more inquiry-oriented opportunities for learning while a lower score is indicative of traditional opportunities for learning. Overall, the treatment and control teachers reported using similar teaching practices.

Figure 5. Teacher and Student Classroom Practices Based on Teacher Logs—Treatment Versus Control



This chart is fully described in Appendix E.

Mathematics Achievement

In this section we present findings related to students' mathematics achievement. First, we present the analysis of the exponential functions assessment. This analysis found a statistically significant difference between assessment scores, with treatment students outperforming control students, on average. We also present qualitative analysis from teacher interviews related to treatment teachers' reflections on how the materials supported students' learning.

Students in Treatment Classrooms Outperformed Students in Control Classrooms on an Assessment of Exponential Functions

The analytic sample comprised 514 students. Prior to the start of the study teachers were randomly assigned into either the treatment or control condition within blocks, with the students nested by teacher (Table 8).

A preliminary analysis considering pre-test differences was conducted to evaluate whether group differences were present between the treatment and control groups prior to the start of implementation. The baseline differences between groups is -0.02 standard deviations, which is considered low (Table 9).

As randomization occurred at the teacher level, two-level mixed effects linear regression was performed to account for nesting of students within teachers (a random effect) and included fixed effects for condition, pre-test, and the randomization block. This analysis found a significant difference between the adjusted means of the treatment and control group post-assessment scores. The effect size is considered medium (g = 0.53; see Figure 6 and Table 10).

Table 8. Total Number of Students and Teachers by Study Condition

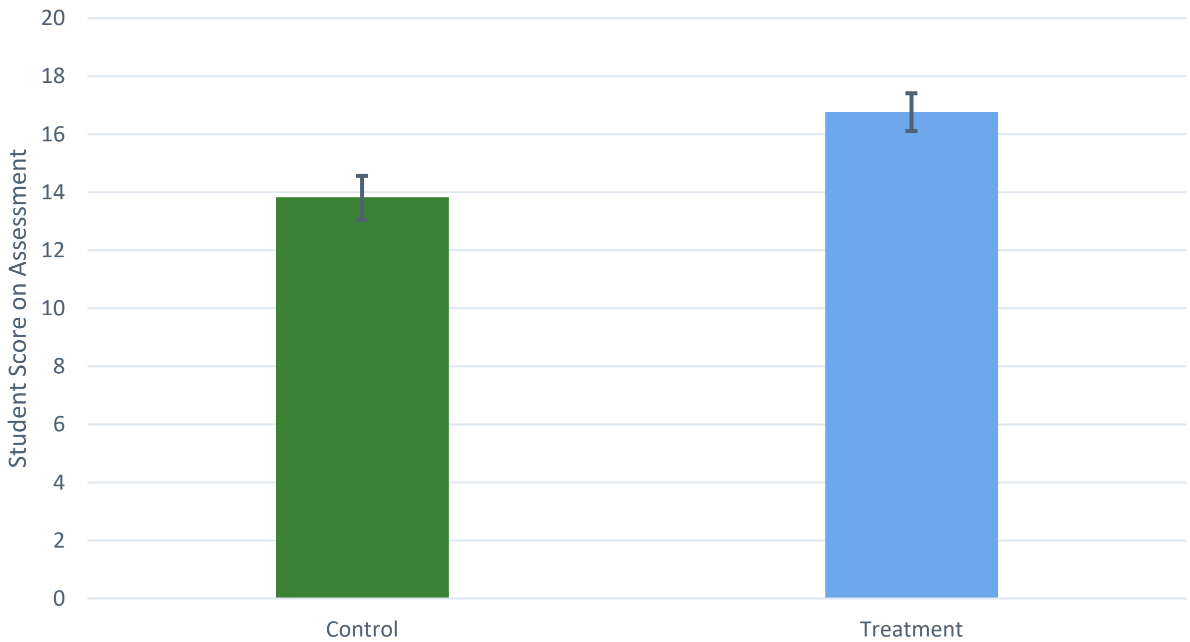
Condition	Teacher *n*	Student *n*
Control	11	252
Treatment	10	262

Table 9: Baseline Equivalence: Pre-Assessment on Exponential Functions

Condition	Mean	*SD*	*N*	Hedges' *g*
Control	10.31	4.69	252	-0.02
Treatment	10.21	3.85	262	

Student assessment data showed a 2.94-point increase in post-test score data between treatment and control students (see Figure 6 and Table 10). In practical terms, students who participated in the treatment condition performed 11 percent better than control students on a researcher-developed measure of exponential functions.

Figure 6. Adjusted Means of Student Exponential Functions Assessment Performance



This image is fully described in Appendix F.

Table 10. Student Exponential Functions Assessment Results

Condition	Adjusted mean	Unadjusted *SD*	*N*	Hedges' *g*
Control	13.81	5.98	252	0.53
Treatment	16.76	5.19	262	

Prisms Lessons Supported Students to Make Connections to Real-World Applications of Mathematics

We interviewed five teachers once they finished teaching their exponential functions unit. They reported that the Prisms module supported students to make connections to real-world applications of mathematics. In this vein, teachers observed that the Prisms lessons motivated students to see mathematics as useful to understanding the world. One teacher put it simply: Prisms is a valuable tool.

> *"Students were more confident to share in the discussion. . . . The student was able to contribute to the conversation because [the IVR component] gave students something specific to talk about."*
>
> *—Middle school teacher*

Prisms Lessons Supported Students' Participation in Whole-Class Mathematics Discussion

Several teachers in our interviews noted that the use of Prisms materials increased students' access to whole-class mathematics discussions. For instance, teachers made the following statements:

- "The math is easier for students to talk about because they have something contextual to go back to."
- "What I love about it so much is it gives you that common point of reference we can always talk about. . . . When talking about growth, decay, factor, and rate, I saw better understanding with growth factor and growth rate."

- "Students continued to make references after the Prisms module. Like the accordion, they talked about how fast the exponential functions take off and would go back to the equation in the Prisms unit."

Overall, teachers reported that the IVR components provided rich contexts that grounded students' mathematical sense-making. The contexts created a concrete scenario that students could refer to as they continued to abstract the mathematical ideas and build their mathematical understanding. Teachers reported that students continually referred to the contexts experiences in the IVR portions and that more students engaged in their whole-class discussions.

Prisms Lessons Created New Opportunities for Students That Regularly Struggle to Feel Successful in Class

Teachers reported that the Prisms lessons created "an even playing ground" that created opportunities for students to feel successful in their mathematics class. Teachers described these new opportunities in two ways.

First, teachers reported that the IVR environment created a safe atmosphere for students to try out their ideas without having to perform for their teacher or peers. Teachers attributed students' feeling of safety to the IVR environment where they could take intellectual risks and to the fact that they were provided multiple opportunities to answer questions. For instance, an Algebra teacher made the following statement:

> *Some students that tend to really struggle, they had successes [in the IVR environment]. . . . I have a handful of students that would have just struggled [in a traditional lesson], but they continually engaged during the [Prisms] lesson.*

Second, teachers stated that students who are not ordinarily recognized as competent in class were able to showcase their knowledge and skills related to the use of the IVR equipment. Teachers hypothesized that students being positioned as competent in this way created a sense of self-efficacy that bolstered their engagement with the Prisms mathematics lesson more generally.

Student Engagement

In this section, we present findings related to student engagement. These findings are related to analysis of teacher and student surveys, as well as teacher interviews.

Student Engagement Survey Revealed No Differences Between Prisms Treatment and Control Groups

The student attitudinal survey was decomposed into two separate subscales: (a) Student Engagement—Mathematical Mindset and (b) Student Engagement—Confidence With Math.

Student Engagement—Mathematical Mindset Subscale

This content includes mathematical mindset, with questions that cover engagement, perseverance, self-efficacy, and utility. There are 16 questions total, and each was rated by students on a 6-point Likert scale that ranged from 1 (*strongly disagree*) to 6 (*strongly agree*). Cronbach's alpha ranges were considered good, ranging between .871 and .875 at pre-test and post-test, respectively.

Researchers conducted baseline equivalence testing to determine whether a difference between treatment and control students' mathematical mindset existed at the start of the study, prior to any interaction with the Prisms content. The baseline differences between groups on pre-test is -0.03 standard deviations, which is considered low (Table 11).

Table 11: Student Engagement—Mathematical Mindset Subscale Baseline Equivalence

Condition	Mean	SD	*N*	Hedges' *g*
Control	4.19	0.71	252	-0.03
Treatment	4.17	0.76	262	

Next, a two-level hierarchical model accounting for nesting of students within teachers (a random effect), including fixed effects for condition, pre-survey, and the randomization block, was conducted to determine whether there were differences between treatment and control groups. This model was not statistically significant (p = 0.96) and indicated that going from the control to the treatment condition did not result in a post-survey change in score (g = 0). In sum, there was no appreciable difference between treatment and control students' self-reports of student engagement and mathematical mindset. Table 12 presents the adjusted means and standard deviations for the Student Engagement—Mathematical Mindset subscale.

Table 12. Student Engagement—Mathematical Mindset Subscale Results

Condition	Adjusted mean	Unadjusted *SD*	*N*	Hedges' *g*
Control	4.08	0.75	252	0
Treatment	4.09	0.83	262	

Student Engagement—Confidence With Math Subscale

This subscale asked students to self-report their confidence level when doing specific mathematical tasks on a 5-point Likert scale that ranged from 1 (*not confident at all*) to 5 (*completely confident*). Seven items in total were included in the subscale. Internal consistency was considered good, with Cronbach's alpha ranging between .862 and .931 at pre-survey and post-survey, respectively.

Preliminary analyses were conducted to evaluate pre-survey equivalence between the treatment and control groups. Table 13 displays these means, standard deviations, and the effect size. The baseline differences between the groups on the pre-survey is 0.05 standard deviations, which is considered within the range of statistical correction. This is accounted for in subsequent analyses.

Table 13. Student Engagement—Confidence With Math Subscale Baseline Equivalence

Condition	Mean	SD	*N*	Hedges' *g*
Control	3.10	0.91	252	0.05
Treatment	3.15	0.96	262	

Finally, a two-level hierarchical model was used to analyze student post-survey outcomes between treatment and control groups. The model accounted for nesting of students within teachers (a random effect) and included the fixed effects for condition, pre-survey, and randomization block. The analysis found no significant difference between the adjusted means of the treatment and control post-survey scores ($p = 0.60$; Table 14).

Table 14. Student Engagement—Confidence With Math Subscale Results

Condition	Adjusted mean	Unadjusted *SD*	*N*	Hedges' *g*
Control	3.51	0.94	252	0.11
Treatment	3.61	0.96	262	

Teachers Reported That Students Were More Engaged During the Prisms Lessons Than Their Regular Mathematics Lessons

Overall, teachers reported increased student engagement as compared to teaching their regular mathematics lessons. Teachers attributed the increased student engagement, partly, to the reasons previously discussed. That is, the contexts experienced in the IVR environments and the ability to take intellectual risks in the IVR environment supported students to engage with the mathematics lessons more than they ordinarily would. In addition, teachers reported that the novelty of the IVR equipment induced students' engagement. For instance, teachers made the following remarks:

- "They [the students] just can't wait to get back in!"
- "Students sparked-up when I told them they would be doing a Prisms activity."
- "Prisms activities were the highlight of some students' days. After a Prisms activity one student remarked, 'What do I have to look forward to the rest of the day?'"

> *"I saw a higher level of engagement using the module rather than class as normal activities."*
>
> *—High school Algebra teacher*

Treatment Students' Survey Related to the Use of Virtual Reality

Overall, students responded positively to the use of IVR to support learning of mathematics. Table 15 summarizes students' responses. More than three-quarters of students agreed with the statements that the IVR lessons helped them to understand mathematics (78.63%), that the use of IVR was worth the time and effort (75.57%), that the IVR lessons made mathematics more interesting (83.21%), and that IVR helped them to see how mathematics is used in the real world (83.97%). Interestingly, a majority of students also agreed with the statement “I learn math better without the IVR activities” (59.16%). Similarly, students did not think that the Prisms lessons could be done just as easily without the IVR equipment; only 45.42 percent of students agreed with such a statement. Taken together, these responses suggest that students found the IVR engaging, meaningful, and supportive of their learning. However, students were less sure of *how* the IVR supported their learning.

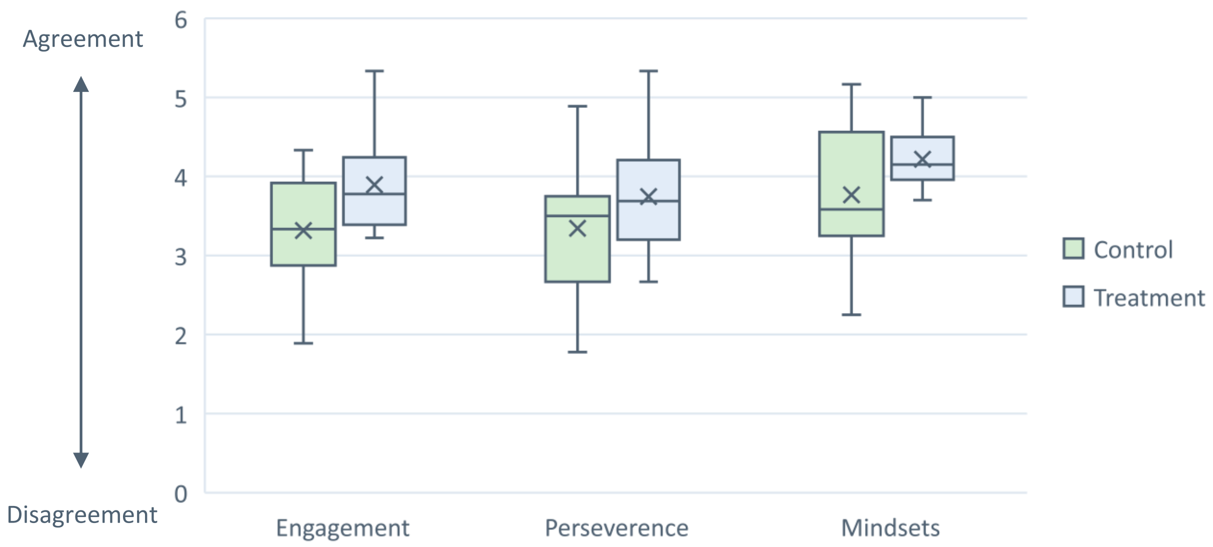
Table 15: Student’s Responses Regarding Use of Virtual Reality

Question	Agree	Disagree
The VR lessons helped me to understand mathematics.	78.63%	21.37%
It is worth the time and effort to use the VR headsets.	75.57%	24.43%
The VR lessons make learning math more interesting to me.	83.21%	16.79%
The VR lessons help me to see how math is used in the real world.	83.97%	16.03%
I learn math better without the VR activities.	59.16%	40.84%
The Prisms math lessons could be done just as easily without VR, like on a computer. (Reverse)	45.42%	54.58%

Teachers Using Prisms Reported Higher Student Engagement Than Control Teachers

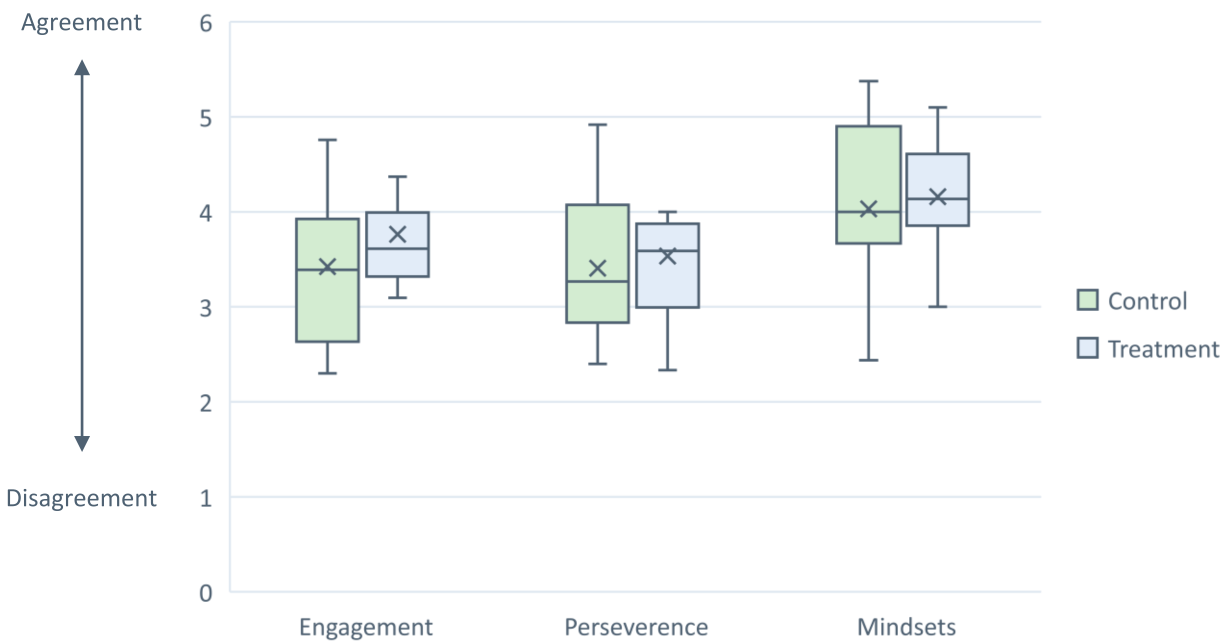
Figures 7 and 8 show the teacher log results of teachers’ perceptions of students’ engagement, perseverance, and mindsets during the Prisms units. The average of each teacher’s response is a single data point for each construct on the plot. For all Prisms units, teachers’ perceptions were consistent with that of the control group (see Figure 8). However, during the exponential functions unit (Figure 7), the treatment teachers reported small but noticeable increases in students’ engagement. Figure 7 reveals that two thirds of treatment teachers reported higher student engagement as compared with half of the control teachers.

Figure 7. Teacher Perceptions of Students' Engagement, Perseverance, and Mindsets (Exponential Functions Unit)



This image is fully described in Appendix G.

Figure 8. Teacher Perceptions of Students' Engagement, Perseverance, and Mindsets (All Prisms Units)



This image is fully described in Appendix H.

Implementation of Prisms Lessons

In this section, we present the qualitative analysis of the implementation of Prisms lesson. We begin with findings related to teachers' self-reported adaptions gathered from the teacher logs. Next, we describe the timing and pacing of the exponential functions lesson based on observations. Finally, we share our analysis of how teachers used the teachers' guide during their enactment of observed lessons.

Teachers Reported Implementing the Lessons With Little to No Adaptations

Treatment teachers reported implementing the IVR days as described in the implementation guide. Of the 70-treatment teacher logs we received, 21 provided teachers' perceptions of how closely their enactment of Prisms lessons followed the implementation guides related to the IVR component and synthesis day component. Nineteen (90%) responses indicated that teachers implemented the IVR days with "little to no changes." One teacher reported on two logs that "many changes" were made to the lesson, but that teacher did not include an elaboration on the nature of those changes.

Teachers reported implementing the synthesis lessons as described in the implementation guide, but to a lesser extent than the IVR components. Fifteen (83%) responses indicated implementing the synthesis lessons with "little to no changes." Three responses (by two teachers) reported "many changes." When teachers reported "many changes," they were asked to share the nature of those changes: one teacher described the omission of the exponential function synthesis lesson, one reported "none," and the other left the section blank.

Teachers reported the following adaptations during the synthesis lesson of the exponential functions. One teacher shared a video from YouTube about water hyacinths before students encountered the related problem on the worksheet. This teacher reported this as a minor change and stated that the video "was a nice lead into the problem they were assigned to do." The teacher who omitted the synthesis lesson entirely shared:

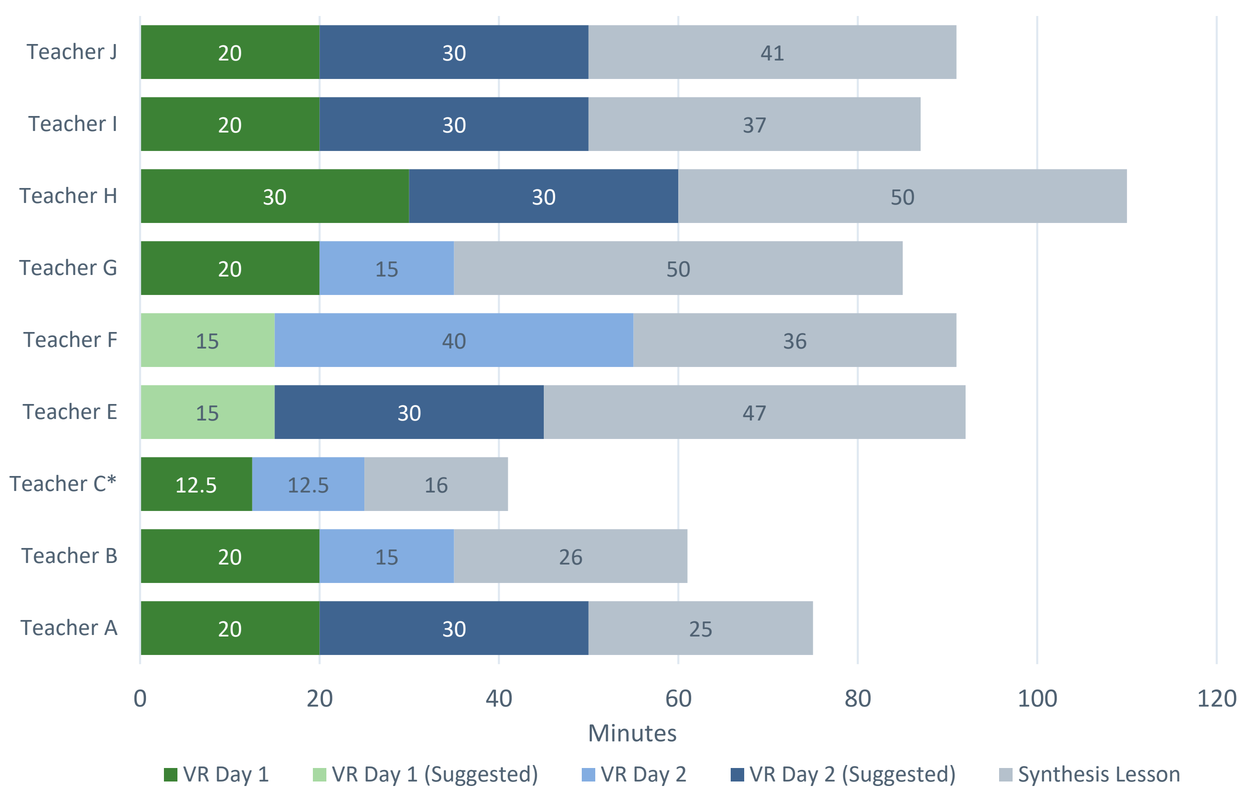
> *I did not use the Prisms synthesis material because I have a lot of resources on my own. I wanted to see if my students would make the connection between the problems we were solving and the problems in the IVR module.*

The Amount of Time Spent on Prisms Lessons Varied by Teacher

We observed nine Prisms exponential functions lessons. We attempted to observe a portion of the virtual reality modules and the synthesis lesson for each teacher in the treatment condition. For the nine lessons that we did observe, we observed portions of the virtual component in eight classrooms and the synthesis lesson in each classroom.

The total amount of time spent on the Prisms lesson was highly variable. The shortest lesson was just over 40 minutes in total, whereas the longest lesson was about 90 minutes. Figure 9 shows how the timing was distributed for each component of the Prisms module. For components that were not observed, we used the suggested timing in Figure 9.

Figure 9: Time Spent on Prisms Lessons



* **The teacher in one lesson we observed implemented both portions of the virtual reality module in one class session. Teacher C directed students to continue with IVR Day 2 immediately after finishing IVR Day 1. As such, it was not evident in the observation when IVR Day 1 ended and IVR Day 2 began. Overall, the IVR portions took 25 minutes in this classroom. For the purposes of this report, we assume time was spent equally between portions (12 minutes each).**

This image is fully described in Appendix I.

IVR Day 1 and Day 2

The mean duration of IVR Day 1 was 20 minutes (n = 7) and ranged from 12 to 30 minutes; the Prisms module overview suggests that IVR Day 1 will require approximately 10–20 minutes of instructional time. The mean duration of IVR Day 2 was also 20 minutes (n = 4) and ranged from 11 to 40 minutes; the Prisms module overview suggests that IVR Day 2 will require approximately 25–35 minutes. Overall, the time spent on the activities for IVR Day 1 were commensurate with the suggested timing; however, the time spent on the activities for IVR Day 2 was slightly shorter than suggested. In three of the four IVR Day 2 lessons we observed, students spent 15 minutes or less in the IVR module.

Synthesis Lesson

The mean duration of the synthesis component was 37 minutes; the suggested timing is between 41 and 54 minutes. The shortest synthesis component was 16 minutes, and the longest was 50 minutes (see Figure 9).

Teachers' Enactment of Lessons Aligned With Teacher Guides

In this section, we first present the enactment of IVR Day 1 by outlining the sequence of the lesson as presented in the teachers guides. Next, we present a summary of IVR Day 2 since we observed fewer lessons than IVR Day 1. Finally, we present how the synthesis day lessons were enacted.

IVR Day 1

The teacher guides present 3 components to the lesson: launch, anticipate, and debrief. We present findings related to each of these components below.

Launch

In the seven lessons we observed, six teachers used the activation questions, or something similar, as provided in the Prisms lesson guide. That is, they introduced IVR Day 1 of the module by stimulating students' background knowledge about the COVID-19 pandemic. In this portion of the lesson, these teachers asked students to recall how the virus spread and the various containment efforts. Students shared their ideas in a whole-class format. Two of these teachers also made references to prior linear functions to highlight that the ensuing context does not have a linear relationship. We observed one teacher who did not frame the experience at all and simply asked the students to begin working on the exponential functions module in their IVR headset. Observations suggest that the activity was not framed before our observation either.

Anticipate

As students completed the IVR module, most teachers circulated around the room to assist students. In the first few minutes, assistance primarily consisted of physical support (e.g., supporting students to put on IVR gear such as head-mounted display and hand controllers) and technical support (e.g., inputting user ID, volume control). While students were in the IVR module, teachers continued to circulate the classroom. One teacher was observed using and referring to the help messages within the IVR platform; we heard students state that it was difficult to use the help feature (which was accessed through a virtual wrist clock). In multiple classes, some students were done with the IVR Day 1 activity before other students had successfully logged in to begin the activity.

Debrief

The debrief for IVR Day 1 generally consisted of the teacher asking what students noticed while they were in the IVR module. We did not observe debrief discussions that focused on the "rate of viral spread" or similar terminology as suggested in the lesson guide; rather, the debrief discussions tended to compare the amount of total spread in each situation without directly attending to the notion of rate. For instance, one teacher summarized the lesson by stating the following:

> So, for the first color [no containment strategy], you noticed the color was getting larger and larger, but each time you did a different one [containment strategy], there was less color for each one.

In another classroom, the notion of rate was brought up in the following interaction during the debrief:

> Teacher: How were [the situations] different?
>
> Student: They spread at different rates.
>
> Teacher: Which one was quickest? [inaudible response] Which one was the slowest?
>
> Student: The one with social distancing and the mask.
>
> Teacher: Great. I want you to keep this in mind when we come back to look at the math.

In this case, the idea of rate was brought forward by a student. The teacher then asked a follow-up question related to rate (e.g., quickest, slowest), but the student was not asked to elaborate or provide evidence that connected two quantities (number of people infected and time) to what was noticed during the module. This demonstrates how the lesson guides were generally followed, but specific teacher moves to bolster particular mathematical understandings were not employed.

IVR Day 2

In the three classrooms that implemented IVR Day 1 and IVR Day 2 on the same day, there was not a debrief between parts; rather the students were directed to continue onto Part 2. We observed four teachers implement IVR Day 2. The debriefs after IVR Day 2 were quick and similar to the IVR Day 1 debriefs in that teachers asked students to describe their experience in the "IVR lab." One teacher expressly discussed the general form for an exponential function and related the equation to a scenario in the lab. This example demonstrates how there is potential for teachers to build mathematical ideas from the virtual experience. However, teachers may need additional support to make the connections visible to their students.

Synthesis Components

The teacher guides present 6 components to the synthesis lesson: frame, launch, work time, discourse, independent practice, and debrief. We present findings related to each of these components below.

Frame

Most teachers framed the synthesis activity by asking students to recall the IVR modules, particularly what they remembered about how the virus spread and comparisons in spread between the containment strategies. Although most teachers provided framing, not all teachers did so by discussing hospital capacity.

Launch

Most teachers presented the launch problem to their students and facilitated the activity as whole-class instruction. This activity was facilitated predominantly in an Initiate-Response-Evaluate (IRE) discourse pattern and consisted of primarily teacher talk. Most teachers used the questions, or similar questions, provided in the lesson guide. As such, teachers highlighted the visual elements (e.g., not being linear) of an exponential function and emphasized the multiplicative relationship, using the term growth factor, while completing the table. Teachers consistently referred to students' experiences with COVID when discussing the exponential function module's exploration of virus spread.

Work Time

Most teachers implemented the work time problem as an extension of the launch. In general, students continued to work on the problems in a whole-class format with the teacher guiding students through the worksheet.

Discourse

This segment of the lesson contained significant departures from the lesson guide. Most teachers did not engage students in the discussion questions as listed in the lesson guide. This is likely because the teachers treated the Work Time Problem as an extension of the launch in a whole-class format rather than as individual work time. Three teachers used one of the prompts listed in the lesson guide. In all three instances, the teachers used the prompt that asked students to consider when new infections will reach 5,000 cases per week. These teachers also referred to the students' real-life experience during COVID.

Independent Practice

Most teachers implemented this lesson as independent practice as suggested in the lesson guide. Most teachers used the "support" section. One teacher used the "support" section for in-class work and assigned the "extend" portion as homework. One teacher used the "solidify" portion of the worksheet. One teacher provided students with different portions based on their perceptions of students' understanding during the IVR component and earlier parts of the synthesis lesson. Students most often completed the independent practice with support from the teacher and our peers, when needed. Teachers tended to circulate the classroom as students completed the worksheet.

Debrief

Five of the nine teachers we observed did not implement this section. This portion was either skipped because there was an alternate class activity or because the class period ended. For those teachers who did implement this section, they posed the question from the lesson guide, or a similar question, that focused students' attention on the distinction between a constant rate of change for a linear function versus a constant growth factor for an exponential function.

Appendices

Appendix A: Student Attitudinal Survey

Construct	1 = *Strongly disagree*; 6 = *Strongly agree*
Engagement	I regularly share my thinking with a small group. I regularly share my thinking with the whole class.
Enjoyment	I like to solve new problems in mathematics. Mathematics is dull and boring. (reverse)
Perseverance	If I get stuck on a math problem, I usually try a different way. I give up easily when confronted with a difficult math problem. (reverse)
Self-Efficacy	I know I can learn the material in my math class. I believe that I can be successful in my math class.
Motivation	I think math class is important. I value math class.

Construct	1 = *Not at all true*; 6 = *Very true*
Mathematics utility	The math that I learn in school is interesting. The math that I learn in school is useful to understanding real-world problems.

Appendix B: Teacher Log

Teachers' perceptions of students'. . .	1 = *Strongly disagree*; 6 = *Strongly agree*
Engagement	My students like to solve real-world problems in mathematics. My students think mathematics is a very interesting subject. A handful of students answered most questions in my class.
Mindsets	Students believe they can be successful in my math class. Students think my math class is useful.
Perseverance	When my students get stuck on a math problem, they usually try to figure out a different way. My students use past experiences to try and understand new math problems. My students give up easily when confronted with a difficult math problem.

Appendix C: Alternative Text Description for Figure 3

Figure 3 is a screenshot ofan example of an independent practice problem included in the Prisms lesson. The problem prompt is:

> Water hyacinth is a very pretty but very invasive plant species that spreads quickly over bodies of water, making it difficult to engage in water-based activities such as boating, fishing, and swimming. Lake Minerva is a small resort town that relies on summer tourism to support the community, but the spread of water hyacinth makes it very difficult to enjoy activities in the lake. Local scientists are studying how quickly water hyacinth is spread over the course of winter and spring so they can help the town prepare the upcoming tourist season. The diagram illustrates their findings.

A bar chart that shows the percent of lake covered in water hyacinth with across three months. Month 0 had 3% coverage, month 1 had 6% coverage, month 2 had 12% coverage, and month 3 had 24% coverage. Students respond to two prompts. The first prompt states: “Use the diagram to fill in the second column in the table to record the percent of Lake Minerva covered in water hyacinth as it spreads each month.” The first column heading is “Month,” with the following values each on a new row: 0, 1, 2, 3. The second column heading is “% of Lake covered in water hyacinth,” with the value 3 in the row for month 0. The remaining cells are empty.

The second prompt states: "What pattern do you see between the output (% of lake covered in water hyacinth) in your table? Describe this pattern with a sentence or two or annotate the table.

Appendix D: Alternative Text Description for Figure 4

Figure 4 is a screenshot of an example of closing debrief problem included in the Prisms teachers' guides. The problem prompt is:

> Today we saw how different exponential models can help us predict what will happen in the future in situations with a constant growth factor. Let's make sure we really understand the difference between exponential and other types of functions before we leave today: A student in another homeroom filled in this table to describe the viral spread in City D. Remember that we said in Week 0 there was 1 new case, and that each week each person spread it to 3 more people.

A table is presented with 2 columns and 5 rows. The first column is labeled "Week" and each cell has the following values: 0, 1, 2, 3, 4. The second column is labeled "Number of New Cases" and each cell has the following values: 1, 3, 6, 9, 12.

The prompt continues:

> Where is the error in the students' reasoning? What might this graph look like? How is this graph different from an exponential graph?

There is a note directed to teachers that states: "Give students a few moments to discuss or jot a response to this question, working alone or in partners. Share exemplar responses (that the student added 3 each week instead of multiplied; the graph would be linear).

Appendix E: Alternative Text Description for Figure 5

Figure 5 presents four box plots related to teachers' and students' perceptions of classroom practices. The scale compares the frequency of inquiry-oriented practices to traditional practices. The y-axis is labeled from 0 (more frequent use of traditional practices) to 6 (more frequent use of inquiry-oriented practices.

Two box plots compare what was reported by teachers. For control teachers, the minimum, median, mean, and maximum are 2.16, 2.54, 2.65, and 3.13, respectively. For treatment teachers the minimum, median, mean, and maximum are 2.29, 2.71, 2.92, and 4.25, respectively.

Two box plots compare what was reported by students. For control students the minimum, median, mean, and maximum are 2.20, 2.65, 2.70, and 3.16, respectively. For treatment students the minimum, median, mean, and maximum are 2.29, 2.89, 2.92, and 4.00, respectively.

Appendix F: Alternative Text Description for Figure 6

Figure 6 is a bar graph with one bar representing the control students adjusted mean score and one bar representing the treatment adjusted mean score. The y-axis is labeled "student score on assessment" and is scaled from 0 to 20.

The control students' adjusted mean is 13.81 with error bar ranging from 13.053 to 14.567. The treatment students' adjusted mean is 16.76 with error bar ranging from 16.11 to 17.41.

Appendix G: Alternative Text Description for Figure 7

Figure 7 presents six box plots related to teachers' perceptions of students' engagement, perseverance, and mindsets during the exponential functions unit. There are 3 sets of boxplots, each set for engagement, perseverance, and mindsets, that compares control teachers' responses with treatment teachers' responses. The y-axis is scaled from 0 (disagreement) to 6 (agreement) related to statements of positive valence with regard to each construct.

Two box plots compare the construct of engagement. For control teachers the minimum, median, mean, and maximum are 1.78, 3.50, 3.34, and 4.89, respectively. For treatment teachers the minimum, median, mean, and maximum are 2.67, 3.69, 3.75, and 5.33, respectively.

Two box plots compare the construct of perseverance. For control teachers the minimum, median, mean, and maximum are 1.89, 3.33, 3.32, and 4.33, respectively. For treatment teachers the minimum, median, mean, and maximum are 3.22, 3.78, 3.89, and 5.33, respectively.

Two box plots compare the construct of mindsets. For control teachers the minimum, median, mean, and maximum are 2.25, 3.58, 3.77, and 5.17, respectively. For treatment teachers the minimum, median, mean, and maximum are 3.70, 4.15, 4.22, and 5.00, respectively.

Appendix H: Alternative Text Description for Figure 8

Figure 8 presents six box plots related to teachers' perceptions of students' engagement, perseverance, and mindsets over the duration of the intervention. There are 3 sets of boxplots, each set for engagement, perseverance, and mindsets, that compares control teachers' responses with treatment teachers' responses. The y-axis is scaled from 0 (disagreement) to 6 (agreement) related to statements of positive valence with regard to each construct.

Two box plots compare the construct of engagement. For control teachers the minimum, median, mean, and maximum are 2.30, 3.89, 3.42, and 4.76, respectively. For treatment teachers the minimum, median, mean, and maximum are 3.10, 3.61, 3.76, and 5.33, respectively.

Two box plots compare the construct of perseverance. For control teachers the minimum, median, mean, and maximum are 2.40, 3.27, 3.41, and 4.92, respectively. For treatment teachers the minimum, median, mean, and maximum are 2.33, 3.59, 3.53, and 5.33, respectively.

Two box plots compare the construct of mindsets. For control teachers the minimum, median, mean, and maximum are 2.44, 4.00, 4.03, and 5.38, respectively. For treatment teachers the minimum, median, mean, and maximum are 3.00, 4.14, 4.16, and 5.10, respectively.

Appendix I: Alternative Text Description for Figure 9

Figure 9 is segmented bar graph that shows the total amount of time in minutes teachers spent on the Prisms exponential functions lesson. Each bar shows how each teacher allocated time for IVR Day 1, IVR Day 2, and the Synthesis Lesson. Suggested times are used when a portion of the lesson was not observed.

- Teacher A: 20 minutes, 30 minutes (suggested), and 25 minutes. Total equals 75 minutes.
- Teacher B: 20 minutes, 15 minutes, and 26 minutes. Total equals 61 minutes.
- Teacher C: 12.5 minutes, 12.5 minutes, and 16 minutes. Total 41 minutes.
- Teacher E: 15 minutes (suggested), 30 minutes (suggested), and 47 minutes. Total 92 minutes.
- Teacher F: 15 minutes (suggested), 40 minutes, and 36 minutes. Total 91 minutes.
- Teacher G: 20 minutes, 15 minutes, and 50 minutes. Total 85 minutes.
- Teacher H: 30 minutes, 30 minutes (suggested), and 50 minutes. Total 110 minutes.
- Teacher I: 20 minutes, 30 minutes (suggested), and 37 minutes. Total 87 minutes.
- Teacher J: 20 minutes, 30 minutes (suggested), and 41 minutes. Total 91 minutes.